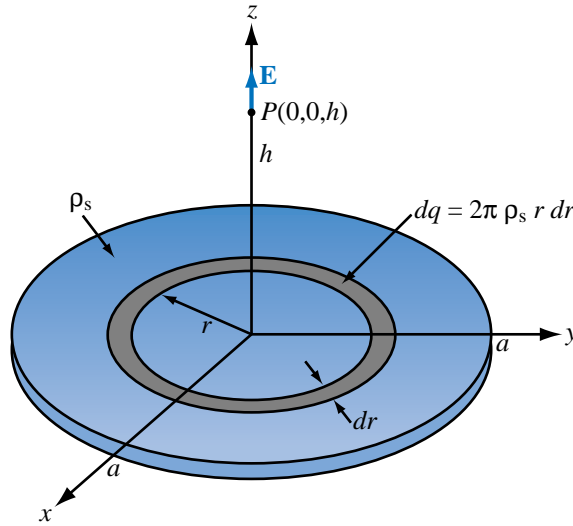


**Problem 4.31** The circular disk of radius  $a$  shown in Fig. 4-7 has uniform charge density  $\rho_s$  across its surface.

- (a) Obtain an expression for the electric potential  $V$  at a point  $P = (0, 0, z)$  on the  $z$ -axis.
- (b) Use your result to find  $\mathbf{E}$  and then evaluate it for  $z = h$ . Compare your final expression with (4.24), which was obtained on the basis of Coulomb's law.

**Solution:**



**Figure P4.31:** Circular disk of charge.

(a) Consider a ring of charge at a radial distance  $r$ . The charge contained in width  $dr$  is

$$dq = \rho_s(2\pi r dr) = 2\pi\rho_s r dr.$$

The potential at  $P$  is

$$dV = \frac{dq}{4\pi\epsilon_0 R} = \frac{2\pi\rho_s r dr}{4\pi\epsilon_0 (r^2 + z^2)^{1/2}}.$$

The potential due to the entire disk is

$$V = \int_0^a dV = \frac{\rho_s}{2\epsilon_0} \int_0^a \frac{r dr}{(r^2 + z^2)^{1/2}} = \frac{\rho_s}{2\epsilon_0} (r^2 + z^2)^{1/2} \Big|_0^a = \frac{\rho_s}{2\epsilon_0} \left[ (a^2 + z^2)^{1/2} - z \right].$$

(b)

$$\mathbf{E} = -\nabla V = -\hat{\mathbf{x}} \frac{\partial V}{\partial x} - \hat{\mathbf{y}} \frac{\partial V}{\partial y} - \hat{\mathbf{z}} \frac{\partial V}{\partial z} = \hat{\mathbf{z}} \frac{\rho_s}{2\epsilon_0} \left[ 1 - \frac{z}{\sqrt{a^2 + z^2}} \right].$$

The expression for  $\mathbf{E}$  reduces to Eq. (4.24) when  $z = h$ .

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