

**Problem 4.32** A circular ring of charge of radius  $a$  lies in the  $x$ - $y$  plane and is centered at the origin. Assume also that the ring is in air and carries a uniform density  $\rho_\ell$ .

(a) Show that the electrical potential at  $(0, 0, z)$  is given by

$$V = \rho_\ell a / [2\epsilon_0 (a^2 + z^2)^{1/2}].$$

(b) Find the corresponding electric field  $\mathbf{E}$ .

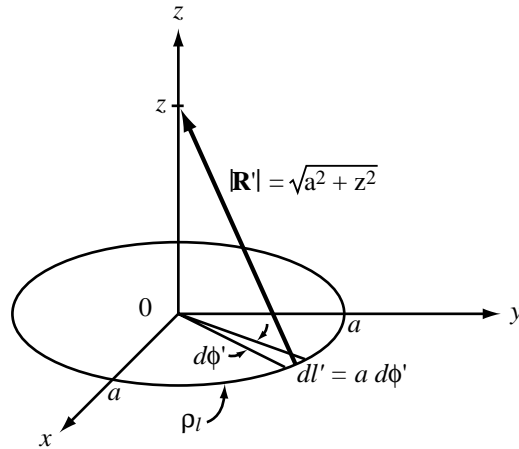
**Solution:**

(a) For the ring of charge shown in Fig. P4.32, using Eq. (3.67) in Eq. (4.48c) gives

$$V(\mathbf{R}) = \frac{1}{4\pi\epsilon_0} \int_{l'} \frac{\rho_l}{R'} dl' = \frac{1}{4\pi\epsilon_0} \int_{\phi'=0}^{2\pi} \frac{\rho_l}{\sqrt{a^2 + r^2 - 2ar \cos(\phi' - \phi) + z^2}} a d\phi'.$$

Point  $(0, 0, z)$  in Cartesian coordinates corresponds to  $(r, \phi, z) = (0, \phi, z)$  in cylindrical coordinates. Hence, for  $r = 0$ ,

$$V(0, 0, z) = \frac{1}{4\pi\epsilon_0} \int_{\phi'=0}^{2\pi} \frac{\rho_l}{\sqrt{a^2 + z^2}} a d\phi' = \frac{\rho_l a}{2\epsilon_0 \sqrt{a^2 + z^2}}.$$



**Figure P4.32:** Ring of charge.

(b) From Eq. (4.51),

$$\mathbf{E} = -\nabla V = -\hat{\mathbf{z}} \frac{\rho_l a}{2\epsilon_0} \frac{\partial}{\partial z} (a^2 + z^2)^{-1/2} = \hat{\mathbf{z}} \frac{\rho_l a}{2\epsilon_0} \frac{z}{(a^2 + z^2)^{3/2}} \quad (\text{V/m}).$$