

Problem 4.33 Show that the electric potential difference V_{12} between two points in air at radial distances r_1 and r_2 from an infinite line of charge with density ρ_ℓ along the z -axis is $V_{12} = (\rho_\ell/2\pi\epsilon_0) \ln(r_2/r_1)$.

Solution: From Eq. (4.33), the electric field due to an infinite line of charge is

$$\mathbf{E} = \hat{\mathbf{r}}E_r = \hat{\mathbf{r}} \frac{\rho_\ell}{2\pi\epsilon_0 r}.$$

Hence, the potential difference is

$$V_{12} = - \int_{r_2}^{r_1} \mathbf{E} \cdot d\mathbf{l} = - \int_{r_2}^{r_1} \frac{\hat{\mathbf{r}}\rho_\ell}{2\pi\epsilon_0 r} \cdot \hat{\mathbf{r}} dr = \frac{\rho_\ell}{2\pi\epsilon_0} \ln\left(\frac{r_2}{r_1}\right).$$
