

Problem 4.48 With reference to Fig. 4-19, find \mathbf{E}_1 if $\mathbf{E}_2 = \hat{\mathbf{x}}3 - \hat{\mathbf{y}}2 + \hat{\mathbf{z}}2$ (V/m), $\epsilon_1 = 2\epsilon_0$, $\epsilon_2 = 18\epsilon_0$, and the boundary has a surface charge density $\rho_s = 3.54 \times 10^{-11}$ (C/m²). What angle does \mathbf{E}_2 make with the z -axis?

Solution: We know that $\mathbf{E}_{1t} = \mathbf{E}_{2t}$ for any 2 media. Hence, $\mathbf{E}_{1t} = \mathbf{E}_{2t} = \hat{\mathbf{x}}3 - \hat{\mathbf{y}}2$. Also, $(\mathbf{D}_1 - \mathbf{D}_2) \cdot \hat{\mathbf{n}} = \rho_s$ (from Table 4.3). Hence, $\epsilon_1(\mathbf{E}_1 \cdot \hat{\mathbf{n}}) - \epsilon_2(\mathbf{E}_2 \cdot \hat{\mathbf{n}}) = \rho_s$, which gives

$$E_{1z} = \frac{\rho_s + \epsilon_2 E_{2z}}{\epsilon_1} = \frac{3.54 \times 10^{-11}}{2\epsilon_0} + \frac{18(2)}{2} = \frac{3.54 \times 10^{-11}}{2 \times 8.85 \times 10^{-12}} + 18 = 20 \quad (\text{V/m}).$$

Hence, $\mathbf{E}_1 = \hat{\mathbf{x}}3 - \hat{\mathbf{y}}2 + \hat{\mathbf{z}}20$ (V/m). Finding the angle \mathbf{E}_2 makes with the z -axis:

$$\mathbf{E}_2 \cdot \hat{\mathbf{z}} = |\mathbf{E}_2| \cos \theta, \quad 2 = \sqrt{9+4+4} \cos \theta, \quad \theta = \cos^{-1} \left(\frac{2}{\sqrt{17}} \right) = 61^\circ.$$
