Problem 4.53 Dielectric breakdown occurs in a material whenever the magnitude of the field **E** exceeds the dielectric strength anywhere in that material. In the coaxial capacitor of Example 4-12,

(a) At what value of r is $|E|$ maximum?

(b) What is the breakdown voltage if $a = 1$ cm, $b = 2$ cm, and the dielectric material is mica with $\varepsilon_r = 6$?

Solution:

(a) From Eq. (4.114), $\mathbf{E} = -\hat{\mathbf{r}}\rho_l/2\pi\varepsilon r$ for $a < r < b$. Thus, it is evident that $|\mathbf{E}|$ is maximum at $r = a$.

(b) The dielectric breaks down when $|\mathbf{E}| > 200$ (MV/m) (see Table 4-2), or

$$|\mathbf{E}| = \frac{\rho_l}{2\pi\varepsilon r} = \frac{\rho_l}{2\pi(6\varepsilon_0)(10^{-2})} = 200 \quad \text{(MV/m)},$$

which gives $\rho_l = (200 \text{ MV/m})(2\pi)6(8.854 \times 10^{-12})(0.01) = 667.6$ (μC/m).

From Eq. (4.115), we can find the voltage corresponding to that charge density,

$$V = \frac{\rho_l}{2\pi\varepsilon} \ln\left(\frac{b}{a}\right) = \frac{(667.6\,\mu\text{C/m})}{12\pi(8.854 \times 10^{-12}\text{ F/m})} \ln(2) = 1.39 \quad \text{(MV)}.$$

Thus, $V = 1.39$ (MV) is the breakdown voltage for this capacitor.