

Problem 4.55 In a dielectric medium with $\epsilon_r = 4$, the electric field is given by

$$\mathbf{E} = \hat{\mathbf{x}}(x^2 + 2z) + \hat{\mathbf{y}}x^2 - \hat{\mathbf{z}}(y + z) \quad (\text{V/m})$$

Calculate the electrostatic energy stored in the region $-1 \text{ m} \leq x \leq 1 \text{ m}$, $0 \leq y \leq 2 \text{ m}$, and $0 \leq z \leq 3 \text{ m}$.

Solution: Electrostatic potential energy is given by Eq. (4.124),

$$\begin{aligned} W_e &= \frac{1}{2} \int_V \epsilon |\mathbf{E}|^2 dV = \frac{\epsilon}{2} \int_{z=0}^3 \int_{y=0}^2 \int_{x=-1}^1 [(x^2 + 2z)^2 + x^4 + (y + z)^2] dx dy dz \\ &= \frac{4\epsilon_0}{2} \left(\left(\left(\frac{2}{5} x^5 y z + \frac{2}{3} z^2 x^3 y + \frac{4}{3} z^3 x y + \frac{1}{12} (y + z)^4 x \right) \right) \bigg|_{x=-1}^1 \right) \bigg|_{y=0}^2 \bigg|_{z=0}^3 \\ &= \frac{4\epsilon_0}{2} \left(\frac{1304}{5} \right) = 4.62 \times 10^{-9} \quad (\text{J}). \end{aligned}$$
