Problem 4.56 Figure P4.56(a) depicts a capacitor consisting of two parallel, conducting plates separated by a distance d. The space between the plates contains two adjacent dielectrics, one with permittivity ε_1 and surface area A_1 and another with ε_2 and A_2. The objective of this problem is to show that the capacitance C of the configuration shown in Fig. P4.56(a) is equivalent to two capacitances in parallel, as illustrated in Fig. P4.56(b), with

$$C = C_1 + C_2 \tag{19}$$

where

$$C_1 = \frac{\varepsilon_1 A_1}{d} \tag{20}$$

$$C_2 = \frac{\varepsilon_2 A_2}{d} \tag{21}$$

To this end, proceed as follows:

(a) Find the electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$ in the two dielectric layers.

(b) Calculate the energy stored in each section and use the result to calculate C_1 and C_2.

(c) Use the total energy stored in the capacitor to obtain an expression for C. Show that (19) is indeed a valid result.

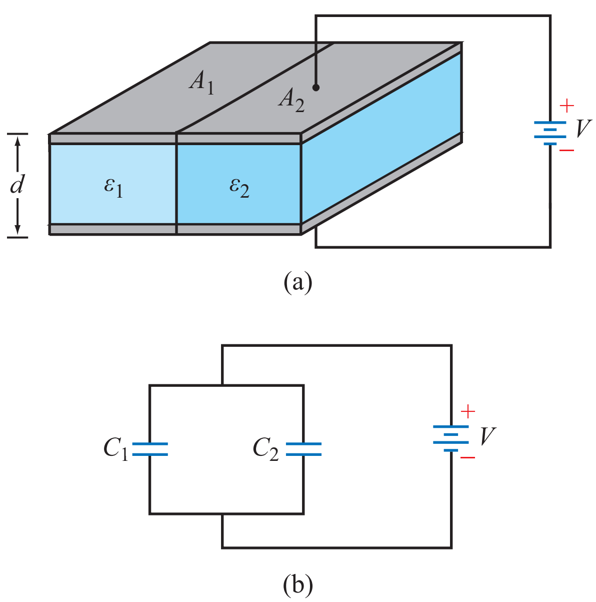
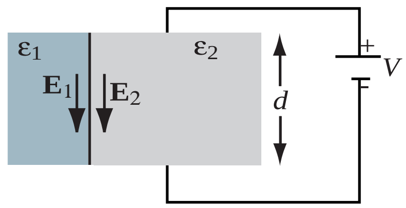


Figure P4.56: (a) Capacitor with parallel dielectric section, and (b) equivalent circuit.

Solution:



(c)

Figure P4.56: (c) Electric field inside of capacitor.

(a) Within each dielectric section, $\mathbf{E}$ will point from the plate with positive voltage to the plate with negative voltage, as shown in Fig. P4-56(c). From $V = Ed$,

$$E_1 = E_2 = \frac{V}{d}.$$

(b)

$$W_{e_1} = \frac{1}{2}\varepsilon_1 E_1^2 \cdot \mathcal{V} = \frac{1}{2}\varepsilon_1 \frac{V^2}{d^2} \cdot A_1 d = \frac{1}{2}\varepsilon_1 V^2 \frac{A_1}{d}.$$

But, from Eq. (4.121),

$$W_{e_1} = \frac{1}{2} C_1 V^2.$$

Hence $C_1 = \varepsilon_1 \dfrac{A_1}{d}$. Similarly, $C_2 = \varepsilon_2 \dfrac{A_2}{d}$.

(c) Total energy is

$$W_e = W_{e_1} + W_{e_2} = \frac{1}{2}\frac{V^2}{d}(\varepsilon_1 A_1 + \varepsilon_2 A_2) = \frac{1}{2} C V^2.$$

Hence,

$$C = \frac{\varepsilon_1 A_1}{d} + \frac{\varepsilon_2 A_2}{d} = C_1 + C_2.$$