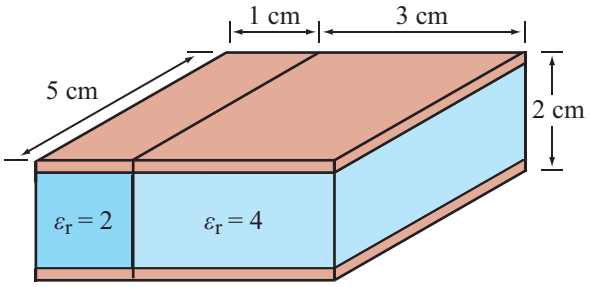
Problem 4.57 Use the result of Problem 4.56 to determine the capacitance for each of the following configurations:

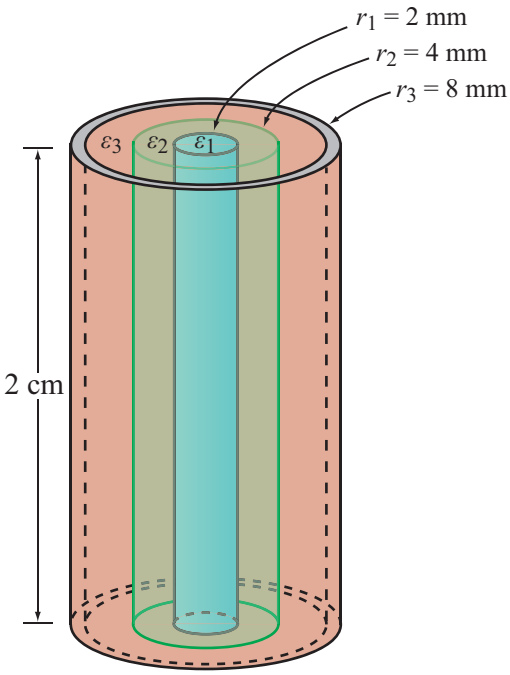
(a) Conducting plates are on top and bottom faces of the rectangular structure in Fig. P4.57(a).

(b) Conducting plates are on front and back faces of the structure in Fig. P4.57(a).

(c) Conducting plates are on top and bottom faces of the cylindrical structure in Fig. P4.57(b).



(a)



$\varepsilon_1 = 8\varepsilon_0$; $\varepsilon_2 = 4\varepsilon_0$; $\varepsilon_3 = 2\varepsilon_0$

(b)

Figure P4.57: Dielectric sections for Problems 4.57 and 4.59.

Solution:

(a) The two capacitors share the same voltage; hence they are in parallel.

$$C_1 = \varepsilon_1 \frac{A_1}{d} = 2\varepsilon_0 \frac{(5\times 1)\times 10^{-4}}{2\times 10^{-2}} = 5\varepsilon_0 \times 10^{-2},$$
$$C_2 = \varepsilon_2 \frac{A_2}{d} = 4\varepsilon_0 \frac{(5\times 3)\times 10^{-4}}{2\times 10^{-2}} = 30\varepsilon_0 \times 10^{-2},$$
$$C = C_1 + C_2 = (5\varepsilon_0 + 30\varepsilon_0)\times 10^{-2} = 0.35\varepsilon_0 = 3.1\times 10^{-12}\ \text{F}.$$

(b)

$$C_1 = \varepsilon_1 \frac{A_1}{d} = 2\varepsilon_0 \frac{(2\times 1)\times 10^{-4}}{5\times 10^{-2}} = 0.8\,\varepsilon_0 \times 10^{-2},$$
$$C_2 = \varepsilon_2 \frac{A_2}{d} = 4\varepsilon_0 \frac{(3\times 2)\times 10^{-4}}{5\times 10^{-2}} = \frac{24}{5}\varepsilon_0 \times 10^{-2},$$
$$C = C_1 + C_2 = 0.5\times 10^{-12}\ \text{F}.$$

(c)

$$C_1 = \varepsilon_1 \frac{A_1}{d} = 8\varepsilon_0 \frac{(\pi r_1^2)}{2\times 10^{-2}} = \frac{4\pi\varepsilon_0}{10^{-2}}(2\times 10^{-3})^2 = 0.04\times 10^{-12}\ \text{F},$$
$$C_2 = \varepsilon_2 \frac{A_2}{d}$$
$$= 4\varepsilon_0 \frac{(\pi(r_2^2 - r_1^2)}{2\times 10^{-2}} = \frac{2\pi\varepsilon_0}{10^{-2}}[(4\times 10^{-3})^2 - (2\times 10^{-3})^2] = 0.06\times 10^{-12}\ \text{F},$$
$$C_3 = \varepsilon_3 \frac{A_3}{d}$$
$$= 2\varepsilon_0 \frac{(\pi(r_3^2 - r_2^2)}{2\times 10^{-2}} = \frac{\pi\varepsilon_0}{10^{-2}}[(8\times 10^{-3})^2 - (4\times 10^{-3})^2] = 0.12\times 10^{-12}\ \text{F},$$
$$C = C_1 + C_2 + C_3 = 0.22\times 10^{-12}\ \text{F}.$$