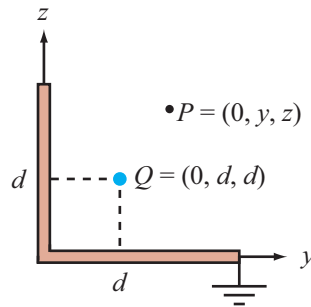


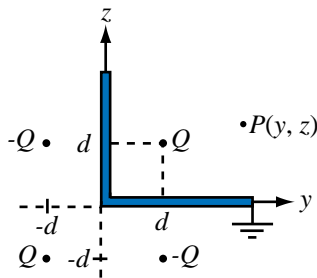
**Problem 4.61** With reference to Fig. P4.61, charge  $Q$  is located at a distance  $d$  above a grounded half-plane located in the  $x$ - $y$  plane and at a distance  $d$  from another grounded half-plane in the  $x$ - $z$  plane. Use the image method to

- (a) Establish the magnitudes, polarities, and locations of the images of charge  $Q$  with respect to each of the two ground planes (as if each is infinite in extent).
- (b) Find the electric potential and electric field at an arbitrary point  $P = (0, y, z)$ .



**Figure P4.61:** Charge  $Q$  next to two perpendicular, grounded, conducting half-planes.

**Solution:**



**Figure P4.61:** (a) Image charges.

(a) The original charge has magnitude and polarity  $+Q$  at location  $(0, d, d)$ . Since the negative  $y$ -axis is shielded from the region of interest, there might as well be a conducting half-plane extending in the  $-y$  direction as well as the  $+y$  direction. This ground plane gives rise to an image charge of magnitude and polarity  $-Q$  at location  $(0, d, -d)$ . In addition, since charges exist on the conducting half plane in the  $+z$  direction, an image of this conducting half plane also appears in the  $-z$  direction.

This ground plane in the  $x$ - $z$  plane gives rise to the image charges of  $-Q$  at  $(0, -d, d)$  and  $+Q$  at  $(0, -d, -d)$ .

(b) Using Eq. (4.47) with  $N = 4$ ,

$$\begin{aligned}
 V(x, y, z) &= \frac{Q}{4\pi\epsilon} \left( \frac{1}{|\hat{\mathbf{x}}x + \hat{\mathbf{y}}(y-d) + \hat{\mathbf{z}}(z-d)|} - \frac{1}{|\hat{\mathbf{x}}x + \hat{\mathbf{y}}(y+d) + \hat{\mathbf{z}}(z-d)|} \right. \\
 &\quad \left. + \frac{1}{|\hat{\mathbf{x}}x + \hat{\mathbf{y}}(y+d) + \hat{\mathbf{z}}(z+d)|} - \frac{1}{|\hat{\mathbf{x}}x + \hat{\mathbf{y}}(y-d) + \hat{\mathbf{z}}(z+d)|} \right) \\
 &= \frac{Q}{4\pi\epsilon} \left( \frac{1}{\sqrt{x^2 + (y-d)^2 + (z-d)^2}} - \frac{1}{\sqrt{x^2 + (y+d)^2 + (z-d)^2}} \right. \\
 &\quad \left. + \frac{1}{\sqrt{x^2 + (y+d)^2 + (z+d)^2}} - \frac{1}{\sqrt{x^2 + (y-d)^2 + (z+d)^2}} \right) \\
 &= \frac{Q}{4\pi\epsilon} \left( \frac{1}{\sqrt{x^2 + y^2 - 2yd + z^2 - 2zd + 2d^2}} \right. \\
 &\quad - \frac{1}{\sqrt{x^2 + y^2 + 2yd + z^2 - 2zd + 2d^2}} \\
 &\quad + \frac{1}{\sqrt{x^2 + y^2 + 2yd + z^2 + 2zd + 2d^2}} \\
 &\quad \left. - \frac{1}{\sqrt{x^2 + y^2 - 2yd + z^2 + 2zd + 2d^2}} \right) \quad (\text{V}).
 \end{aligned}$$

From Eq. (4.51),

$$\begin{aligned}
 \mathbf{E} &= -\nabla V \\
 &= \frac{Q}{4\pi\epsilon} \left( \nabla \frac{1}{\sqrt{x^2 + (y-d)^2 + (z-d)^2}} - \nabla \frac{1}{\sqrt{x^2 + (y+d)^2 + (z-d)^2}} \right. \\
 &\quad \left. + \nabla \frac{1}{\sqrt{x^2 + (y+d)^2 + (z+d)^2}} - \nabla \frac{1}{\sqrt{x^2 + (y-d)^2 + (z+d)^2}} \right) \\
 &= \frac{Q}{4\pi\epsilon} \left( \frac{\hat{\mathbf{x}}x + \hat{\mathbf{y}}(y-d) + \hat{\mathbf{z}}(z-d)}{(x^2 + (y-d)^2 + (z-d)^2)^{3/2}} - \frac{\hat{\mathbf{x}}x + \hat{\mathbf{y}}(y+d) + \hat{\mathbf{z}}(z-d)}{(x^2 + (y+d)^2 + (z-d)^2)^{3/2}} \right. \\
 &\quad \left. + \frac{\hat{\mathbf{x}}x + \hat{\mathbf{y}}(y+d) + \hat{\mathbf{z}}(z+d)}{(x^2 + (y+d)^2 + (z+d)^2)^{3/2}} - \frac{\hat{\mathbf{x}}x + \hat{\mathbf{y}}(y-d) + \hat{\mathbf{z}}(z+d)}{(x^2 + (y-d)^2 + (z+d)^2)^{3/2}} \right) \quad (\text{V/m}).
 \end{aligned}$$