

Problem 4.9 A circular beam of charge of radius a consists of electrons moving with a constant speed u along the $+z$ -direction. The beam's axis is coincident with the z -axis and the electron charge density is given by

$$\rho_v = -cr^2 \quad (\text{C/m}^3)$$

where c is a constant and r is the radial distance from the axis of the beam.

- (a) Determine the charge density per unit length.
- (b) Determine the current crossing the z -plane.

Solution:

(a)

$$\begin{aligned} \rho_l &= \int \rho_v \, ds \\ &= \int_{r=0}^a \int_{\phi=0}^{2\pi} -cr^2 \cdot r \, dr \, d\phi = -2\pi c \left. \frac{r^4}{4} \right|_0^a = -\frac{\pi c a^4}{2} \quad (\text{C/m}). \end{aligned}$$

(b)

$$\begin{aligned} \mathbf{J} &= \rho_v \mathbf{u} = -\hat{\mathbf{z}} cr^2 u \quad (\text{A/m}^2) \\ I &= \int \mathbf{J} \cdot d\mathbf{s} \\ &= \int_{r=0}^a \int_{\phi=0}^{2\pi} (-\hat{\mathbf{z}} cur^2) \cdot \hat{\mathbf{z}} r \, dr \, d\phi \\ &= -2\pi cu \int_0^a r^3 \, dr = -\frac{\pi c u a^4}{2} = \rho_l u. \quad (\text{A}). \end{aligned}$$
