Problem 5.14 Two parallel, circular loops carrying a current of 40 A each are arranged as shown in Fig. P5.14. The first loop is situated in the x–y plane with its center at the origin, and the second loop's center is at $z = 2$ m. If the two loops have the same radius $a = 3$ m, determine the magnetic field at:

(a) $z = 0$

(b) $z = 1$ m

(c) $z = 2$ m

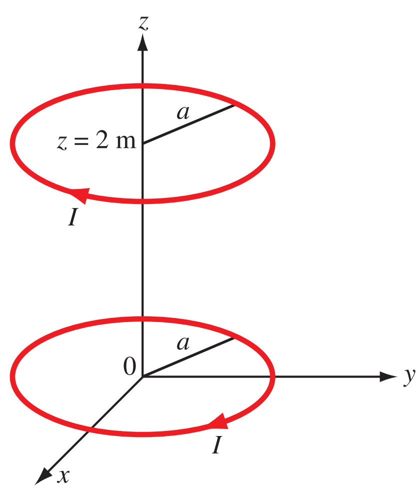

Figure P5.14: Parallel circular loops of Problem 5.14.

Solution: The magnetic field due to a circular loop is given by (5.34) for a loop in the x–y plane carrying a current I in the $+\hat{\boldsymbol{\phi}}$-direction. Considering that the bottom loop in Fig. is in the x–y plane, but the current direction is along $-\hat{\boldsymbol{\phi}}$,

$$\mathbf{H}_1 = -\hat{\mathbf{z}} \frac{Ia^2}{2(a^2 + z^2)^{3/2}} ,$$

where z is the observation point along the z-axis. For the second loop, which is at a height of 2 m, we can use the same expression but z should be replaced with $(z-2)$. Hence,

$$\mathbf{H}_2 = -\hat{\mathbf{z}} \frac{Ia^2}{2[a^2 + (z-2)^2]^{3/2}} .$$

The total field is

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2 = -\hat{\mathbf{z}} \frac{Ia^2}{2} \left[\frac{1}{(a^2+z^2)^{3/2}} + \frac{1}{[a^2+(z-2)^2]^{3/2}} \right] \text{ A/m.}$$

(a) At $z = 0$, and with $a = 3$ m and $I = 40$ A,

$$\mathbf{H} = -\hat{\mathbf{z}}\,\frac{40 \times 9}{2}\left[\frac{1}{3^3} + \frac{1}{(9+4)^{3/2}}\right] = -\hat{\mathbf{z}}\,10.5 \text{ A/m}.$$

(b) At $z = 1$ m (midway between the loops):

$$\mathbf{H} = -\hat{\mathbf{z}}\,\frac{40 \times 9}{2}\left[\frac{1}{(9+1)^{3/2}} + \frac{1}{(9+1)^{3/2}}\right] = -\hat{\mathbf{z}}\,11.38 \text{ A/m}.$$

(c) At $z = 2$ m, $\mathbf{H}$ should be the same as at $z = 0$. Thus,

$$\mathbf{H} = -\hat{\mathbf{z}}\,10.5 \text{ A/m}.$$