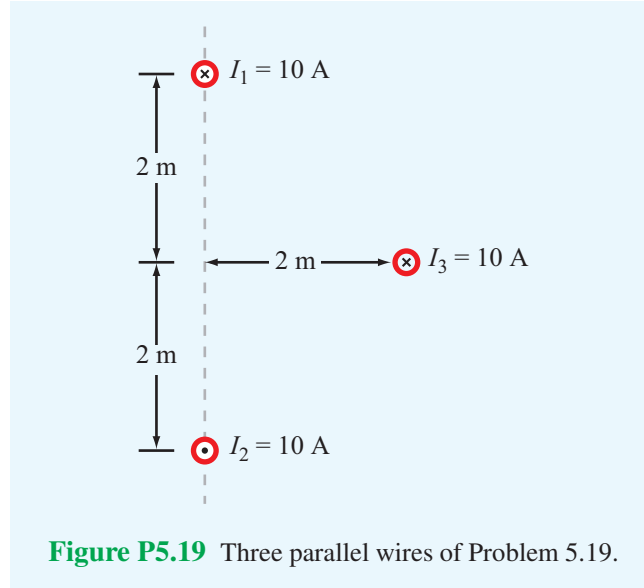


**5.19** Three long, parallel wires are arranged as shown in Fig. P5.19. Determine the force per unit length acting on the wire carrying  $I_3$ .



**Figure P5.19** Three parallel wires of Problem 5.19.

**Solution:** Since  $I_1$  and  $I_2$  are equal in magnitude and opposite in direction, and equidistant from  $I_3$ , our intuitive answer might be that the net force on  $I_3$  is zero. As we will see, that's not the correct answer. The field due to  $I_1$  (which is along  $\hat{\mathbf{y}}$ ) at location of  $I_3$  is

$$\mathbf{B}_1 = \hat{\mathbf{b}}_1 \frac{\mu_0 I_1}{2\pi R_1}$$

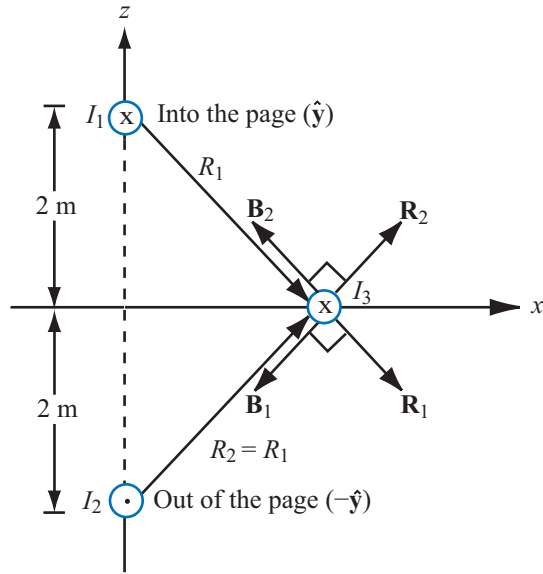
where  $\hat{\mathbf{b}}_1$  is the unit vector in the direction of  $\mathbf{B}_1$  shown in the figure, which is perpendicular to  $\hat{\mathbf{R}}_1$ . The force per unit length exerted on  $I_3$  is

$$\mathbf{F}'_{31} = \frac{\mu_0 I_1 I_3}{2\pi R_1} (\hat{\mathbf{y}} \times \hat{\mathbf{b}}_1) = -\hat{\mathbf{R}}_1 \frac{\mu_0 I_1 I_3}{2\pi R_1}.$$

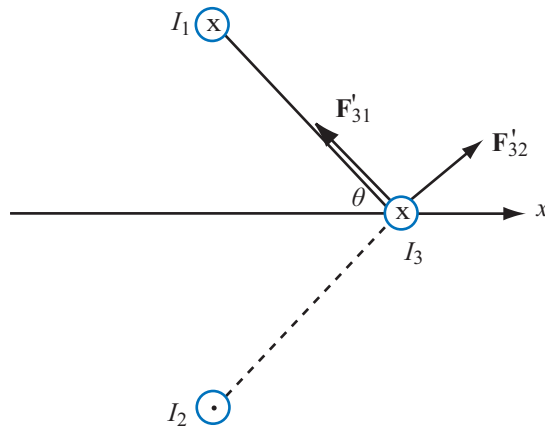
Similarly, the force per unit length excited on  $I_3$  by the field due to  $I_2$  (which is along  $-\hat{\mathbf{y}}$ ) is

$$\mathbf{F}'_{32} = \hat{\mathbf{R}}_2 \frac{\mu_0 I_2 I_3}{2\pi R_2}.$$

The two forces have opposite components along  $\hat{\mathbf{x}}$  and equal components along  $\hat{\mathbf{z}}$ .



**Figure P5.19** (a)  $\mathbf{B}$  fields due to  $I_1$  and  $I_2$  at location of  $I_3$ .



**Figure P5.19** (b) Forces acting on  $I_3$ .

Hence, with  $R_1 = R_2 = \sqrt{8} \text{ m}$  and  $\theta = \sin^{-1}(2/\sqrt{8}) = \sin^{-1}(1/\sqrt{2}) = 45^\circ$ ,

$$\begin{aligned} \mathbf{F}'_3 &= \mathbf{F}'_{31} + \mathbf{F}'_{32} = \hat{\mathbf{z}} \left( \frac{\mu_0 I_1 I_3}{2\pi R_1} + \frac{\mu_0 I_2 I_3}{2\pi R_2} \right) \sin \theta \\ &= \hat{\mathbf{z}} 2 \left( \frac{4\pi \times 10^{-7} \times 10 \times 10}{2\pi \times \sqrt{8}} \right) \times \frac{1}{\sqrt{2}} = \hat{\mathbf{z}} 1 \times 10^{-5} \text{ N/m}. \end{aligned}$$