**Problem 5.21** Current $I$ flows along the positive $z$-direction in the inner conductor of a long coaxial cable and returns through the outer conductor. The inner conductor has radius $a$, and the inner and outer radii of the outer conductor are $b$ and $c$, respectively.

**(a)** Determine the magnetic field in each of the following regions: $0 \le r \le a$, $a \le r \le b$, $b \le r \le c$, and $r \ge c$.

**(b)** Plot the magnitude of $\mathbf{H}$ as a function of $r$ over the range from $r = 0$ to $r = 10$ cm, given that $I = 10$ A, $a = 2$ cm, $b = 4$ cm, and $c = 5$ cm.

**Solution:**

**(a)** Following the solution to Example 5-5, the magnetic field in the region $r < a$,

$$\mathbf{H} = \hat{\boldsymbol{\phi}} \frac{rI}{2\pi a^2},$$

and in the region $a < r < b$,

$$\mathbf{H} = \hat{\boldsymbol{\phi}} \frac{I}{2\pi r}.$$

The total area of the outer conductor is $A = \pi(c^2 - b^2)$ and the fraction of the area of the outer conductor enclosed by a circular contour centered at $r = 0$ in the region $b < r < c$ is

$$\frac{\pi(r^2 - b^2)}{\pi(c^2 - b^2)} = \frac{r^2 - b^2}{c^2 - b^2}.$$

The total current enclosed by a contour of radius $r$ is therefore

$$I_{\text{enclosed}} = I\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right) = I\frac{c^2 - r^2}{c^2 - b^2},$$

and the resulting magnetic field is

$$\mathbf{H} = \hat{\boldsymbol{\phi}} \frac{I_{\text{enclosed}}}{2\pi r} = \hat{\boldsymbol{\phi}} \frac{I}{2\pi r}\left(\frac{c^2 - r^2}{c^2 - b^2}\right).$$

For $r > c$, the total enclosed current is zero: the total current flowing on the inner conductor is equal to the total current flowing on the outer conductor, but they are flowing in opposite directions. Therefore, $\mathbf{H} = 0$.

**(b)** See Fig. P5.21.

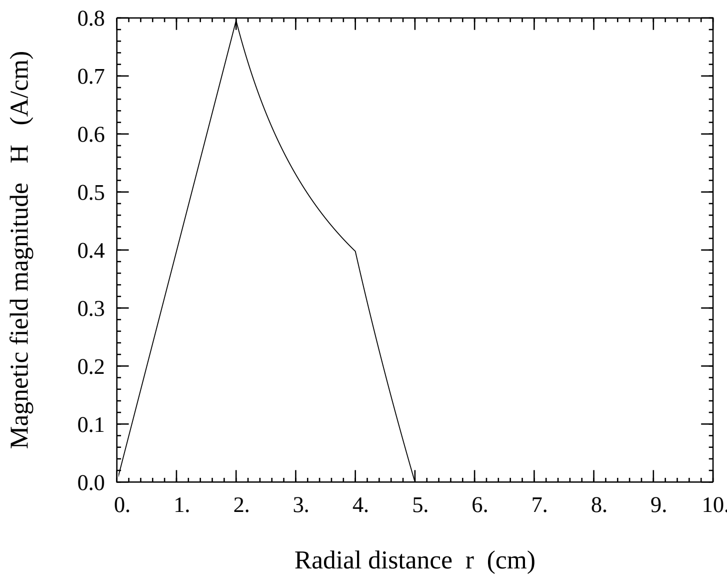


**Figure P5.21:** Problem 5.21.