

Problem 5.23 Repeat Problem 5.22 for a current density $\mathbf{J} = \hat{\mathbf{z}}J_0e^{-r}$.

Solution:

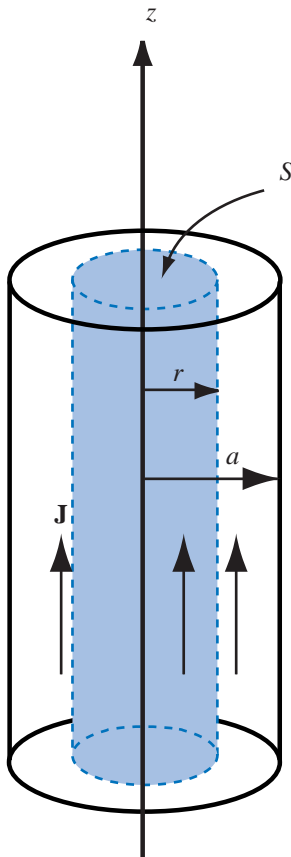


Figure P5.23: Cylindrical current.

(a) For $r \leq a$, Ampère's law is

$$\begin{aligned}\oint_c \mathbf{H} \cdot d\mathbf{l} &= I = \int_S \mathbf{J} \cdot d\mathbf{s}, \\ \hat{\phi} H \cdot \hat{\phi} 2\pi r &= \int_0^r \mathbf{J} \cdot d\mathbf{s} = \int_0^r \hat{\mathbf{z}} J_0 e^{-r} \cdot \hat{\mathbf{z}} 2\pi r dr, \\ 2\pi r H &= 2\pi J_0 \int_0^r r e^{-r} dr \\ &= 2\pi J_0 [-e^{-r}(r+1)]_0^r = 2\pi J_0 [1 - e^{-r}(r+1)].\end{aligned}$$

Hence,

$$\mathbf{H} = \hat{\phi} H = \hat{\phi} \frac{J_0}{r} [1 - e^{-r}(r+1)], \quad \text{for } r \leq a.$$

(b) For $r \geq a$,

$$2\pi r H = 2\pi J_0 [-e^{-r}(r+1)]_0^a = 2\pi J_0 [1 - e^{-a}(a+1)],$$

$$\mathbf{H} = \hat{\phi} H = \hat{\phi} \frac{J_0}{r} [1 - e^{-a}(a+1)], \quad r \geq a.$$
