

**Problem 5.25** A cylindrical conductor whose axis is coincident with the  $z$ -axis has an internal magnetic field given by

$$\mathbf{H} = \hat{\phi} \frac{2}{r} [1 - (4r + 1)e^{-4r}] \quad (\text{A/m}) \quad \text{for } r \leq a$$

where  $a$  is the conductor's radius. If  $a = 5$  cm, what is the total current flowing in the conductor?

**Solution:** We can follow either of two possible approaches. The first involves the use of Ampère's law and the second one involves finding  $\mathbf{J}$  from  $\mathbf{H}$  and then  $\mathbf{I}$  from  $\mathbf{J}$ . We will demonstrate both.

### Approach 1: Ampère's law

Applying Ampère's law at  $r = a$ ,

$$\begin{aligned} \oint_C \mathbf{H} \cdot d\boldsymbol{\ell} \Big|_{r=a} &= I \\ \int_0^{2\pi} \hat{\phi} \frac{2}{r} [1 - (4r + 1)e^{-4r}] \cdot \hat{\phi} r d\phi \Big|_{r=a} &= I \\ I &= 4\pi [1 - (4a + 1)e^{-4a}] \quad (\text{A}). \end{aligned}$$

For  $a = 5$  cm,  $I = 0.22$  (A).

### Approach 2: $\mathbf{H} \rightarrow \mathbf{J} \rightarrow I$

$$\begin{aligned} \mathbf{J} &= \nabla \times \mathbf{H} \\ &= \hat{\mathbf{z}} \frac{1}{r} \frac{\partial}{\partial r} (r H_\phi) \\ &= \hat{\mathbf{z}} \frac{1}{r} \frac{\partial}{\partial r} (2[1 - (4r + 1)e^{-4r}]) \\ &= \hat{\mathbf{z}} \frac{1}{r} [-8e^{-4r} + 8(4r + 1)e^{-4r}] \\ &= \hat{\mathbf{z}} 32e^{-4r}. \\ I &= \int_S \mathbf{J} \cdot d\mathbf{s} = \int_{r=0}^a \hat{\mathbf{z}} 32e^{-4r} \cdot \hat{\mathbf{z}} 2\pi r dr \\ &= 64\pi \int_{r=0}^a r e^{-4r} dr \\ &= \frac{64\pi}{16} [1 - (4a + 1)e^{-4a}] \\ &= 4\pi [1 - (4a + 1)e^{-4a}] \quad (\text{A}). \end{aligned}$$


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