

Problem 5.29 A thin current element extending between $z = -L/2$ and $z = L/2$ carries a current I along $+\hat{\mathbf{z}}$ through a circular cross-section of radius a .

- (a) Find \mathbf{A} at a point P located very far from the origin (assume R is so much larger than L that point P may be considered to be at approximately the same distance from every point along the current element).
- (b) Determine the corresponding \mathbf{H} .

Solution:

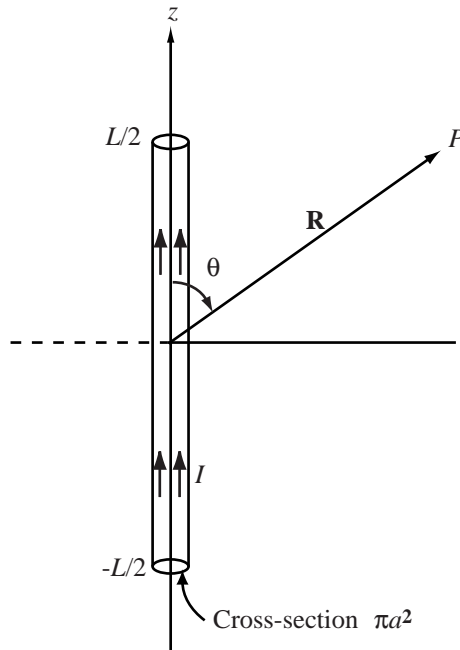


Figure P5.29: Current element of length L observed at distance $R \gg L$.

(a) Since $R \gg L$, we can assume that P is approximately equidistant from all segments of the current element. Hence, with R treated as constant, (5.65) gives

$$\mathbf{A} = \frac{\mu_0}{4\pi} \int_{\mathcal{V}'} \frac{\mathbf{J}}{R'} d\mathcal{V}' = \frac{\mu_0}{4\pi R} \int_{\mathcal{V}'} \hat{\mathbf{z}} \frac{I}{(\pi a^2)} \pi a^2 dz = \hat{\mathbf{z}} \frac{\mu_0 I}{4\pi R} \int_{-L/2}^{L/2} dz = \hat{\mathbf{z}} \frac{\mu_0 IL}{4\pi R}.$$

(b)

$$\begin{aligned}\mathbf{H} &= \frac{1}{\mu_0} \nabla \times \mathbf{A} \\ &= \frac{1}{\mu_0} \left[\hat{\mathbf{x}} \frac{\partial A_z}{\partial y} - \hat{\mathbf{y}} \frac{\partial A_z}{\partial x} \right] \\ &= \frac{1}{\mu_0} \left\{ \hat{\mathbf{x}} \frac{\partial}{\partial y} \left[\frac{\mu_0 I L}{4\pi} \left(\frac{1}{\sqrt{x^2 + y^2 + z^2}} \right) \right] - \hat{\mathbf{y}} \frac{\partial}{\partial x} \left[\frac{\mu_0 I L}{4\pi} \left(\frac{1}{\sqrt{x^2 + y^2 + z^2}} \right) \right] \right\} \\ &= \frac{I L}{4\pi} \left[\frac{-\hat{\mathbf{x}} y + \hat{\mathbf{y}} x}{(x^2 + y^2 + z^2)^{3/2}} \right].\end{aligned}$$
