

Problem 5.32 The x - y plane separates two magnetic media with magnetic permeabilities μ_1 and μ_2 (Fig. P5.32). If there is no surface current at the interface and the magnetic field in medium 1 is

$$\mathbf{H}_1 = \hat{\mathbf{x}}H_{1x} + \hat{\mathbf{y}}H_{1y} + \hat{\mathbf{z}}H_{1z}$$

find:

- (a) \mathbf{H}_2
- (b) θ_1 and θ_2
- (c) Evaluate \mathbf{H}_2 , θ_1 , and θ_2 for $H_{1x} = 2$ (A/m), $H_{1y} = 0$, $H_{1z} = 4$ (A/m), $\mu_1 = \mu_0$, and $\mu_2 = 4\mu_0$

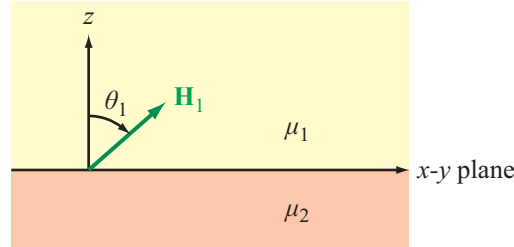


Figure P5.32: Adjacent magnetic media (Problem 5.32).

Solution:

- (a) From (5.80),

$$\mu_1 H_{1n} = \mu_2 H_{2n},$$

and in the absence of surface currents at the interface, (5.85) states

$$H_{1t} = H_{2t}.$$

In this case, $H_{1z} = H_{1n}$, and H_{1x} and H_{1y} are tangential fields. Hence,

$$\mu_1 H_{1z} = \mu_2 H_{2z},$$

$$H_{1x} = H_{2x},$$

$$H_{1y} = H_{2y},$$

and

$$\mathbf{H}_2 = \hat{\mathbf{x}}H_{1x} + \hat{\mathbf{y}}H_{1y} + \hat{\mathbf{z}}\frac{\mu_1}{\mu_2}H_{1z}.$$

(b)

$$\begin{aligned}H_{1t} &= \sqrt{H_{1x}^2 + H_{1y}^2}, \\ \tan \theta_1 &= \frac{H_{1t}}{H_{1z}} = \frac{\sqrt{H_{1x}^2 + H_{1y}^2}}{H_{1z}}, \\ \tan \theta_2 &= \frac{H_{2t}}{H_{2z}} = \frac{\sqrt{H_{1x}^2 + H_{1y}^2}}{\frac{\mu_1}{\mu_2} H_{1z}} = \frac{\mu_2}{\mu_1} \tan \theta_1.\end{aligned}$$

(c)

$$\begin{aligned}\mathbf{H}_2 &= \hat{\mathbf{x}} 2 + \hat{\mathbf{z}} \frac{1}{4} \cdot 4 = \hat{\mathbf{x}} 2 + \hat{\mathbf{z}} \quad (\text{A/m}), \\ \theta_1 &= \tan^{-1} \left(\frac{2}{4} \right) = 26.56^\circ, \\ \theta_2 &= \tan^{-1} \left(\frac{2}{1} \right) = 63.44^\circ.\end{aligned}$$
