Problem 5.34 In Fig. P5.34, the plane defined by $x - y = 1$ separates medium 1 of permeability μ_1 from medium 2 of permeability μ_2. If no surface current exists on the boundary and

$$\mathbf{B}_1 = \hat{\mathbf{x}}2 + \hat{\mathbf{y}}3 \quad \text{(T)}$$

find $\mathbf{B}_2$ and then evaluate your result for $\mu_1 = 5\mu_2$.

Hint: Start by deriving the equation for the unit vector normal to the given plane.

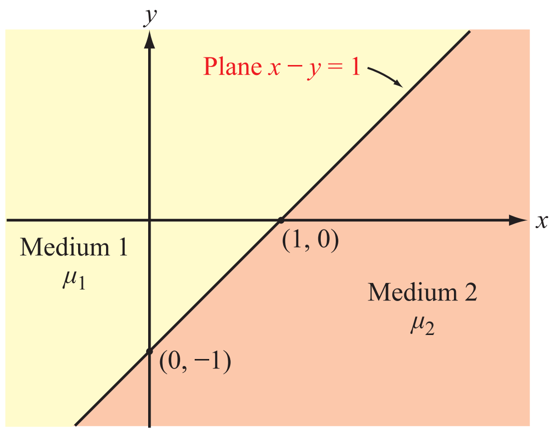

Figure P5.34: Magnetic media separated by the plane $x - y = 1$ (Problem 5.34).

Solution: We need to find $\hat{\mathbf{n}}_2$. To do so, we start by finding any two vectors in the plane $x - y = 1$, and to do that, we need three non-collinear points in that plane. We choose $(0,-1,0)$, $(1,0,0)$, and $(1,0,1)$.

Vector $\mathbf{A}_1$ is from $(0,-1,0)$ to $(1,0,0)$:

$$\mathbf{A}_1 = \hat{\mathbf{x}}1 + \hat{\mathbf{y}}1.$$

Vector $\mathbf{A}_2$ is from $(1,0,0)$ to $(1,0,1)$:

$$\mathbf{A}_2 = \hat{\mathbf{z}}1.$$

Hence, if we take the cross product $\mathbf{A}_2 \times \mathbf{A}_1$, we end up in a direction normal to the given plane, from medium 2 to medium 1,

$$\hat{\mathbf{n}}_2 = \frac{\mathbf{A}_2 \times \mathbf{A}_1}{|\mathbf{A}_2 \times \mathbf{A}_1|} = \frac{\hat{\mathbf{z}}1 \times (\hat{\mathbf{x}}1 + \hat{\mathbf{y}}1)}{|\mathbf{A}_2 \times \mathbf{A}_1|} = \frac{\hat{\mathbf{y}}1 - \hat{\mathbf{x}}1}{\sqrt{1+1}} = \frac{\hat{\mathbf{y}}}{\sqrt{2}} - \frac{\hat{\mathbf{x}}}{\sqrt{2}}.$$

In medium 1, normal component is

$$B_{1n} = \hat{\mathbf{n}}_2 \cdot \mathbf{B}_1 = \left(\frac{\hat{\mathbf{y}}}{\sqrt{2}} - \frac{\hat{\mathbf{x}}}{\sqrt{2}}\right) \cdot (\hat{\mathbf{x}}2 + \hat{\mathbf{y}}3) = \frac{3}{\sqrt{2}} - \frac{2}{\sqrt{2}} = \frac{1}{\sqrt{2}},$$

$$\mathbf{B}_{1n} = \hat{\mathbf{n}}_2 B_{1n} = \left(\frac{\hat{\mathbf{y}}}{\sqrt{2}} - \frac{\hat{\mathbf{x}}}{\sqrt{2}}\right) \cdot \frac{1}{\sqrt{2}} = \frac{\hat{\mathbf{y}}}{2} - \frac{\hat{\mathbf{x}}}{2}.$$

Tangential component is

$$\mathbf{B}_{1t} = \mathbf{B}_1 - \mathbf{B}_{1n} = (\hat{\mathbf{x}}2 + \hat{\mathbf{y}}3) - \left(\frac{\hat{\mathbf{y}}}{2} - \frac{\hat{\mathbf{x}}}{2}\right) = \hat{\mathbf{x}}2.5 + \hat{\mathbf{y}}2.5.$$

Boundary conditions:

$$B_{1n} = B_{2n}, \quad \text{or} \quad \mathbf{B}_{2n} = \frac{\hat{\mathbf{y}}}{2} - \frac{\hat{\mathbf{x}}}{2},$$
$$H_{1t} = H_{2t}, \quad \text{or} \quad \frac{B_{2t}}{\mu_2} = \frac{B_{1t}}{\mu_1}.$$

Hence,

$$\mathbf{B}_{2t} = \frac{\mu_2}{\mu_1}\mathbf{B}_{1t} = \frac{\mu_2}{\mu_1}(\hat{\mathbf{x}}2.5 + \hat{\mathbf{y}}2.5).$$

Finally,

$$\mathbf{B}_2 = \mathbf{B}_{2n} + \mathbf{B}_{2t} = \left(\frac{\hat{\mathbf{y}}}{2} - \frac{\hat{\mathbf{x}}}{2}\right) + \frac{\mu_2}{\mu_1}(\hat{\mathbf{x}}2.5 + \hat{\mathbf{y}}2.5).$$

For $\mu_1 = 5\mu_2$,

$$\mathbf{B}_2 = \hat{\mathbf{y}} \quad \text{(T)}.$$