Problem 5.37 Obtain an expression for the self-inductance per unit length for the parallel wire transmission line of Fig. 5-27(a) in terms of a, d, and μ, where a is the radius of the wires, d is the axis-to-axis distance between the wires, and μ is the permeability of the medium in which they reside.

Solution:

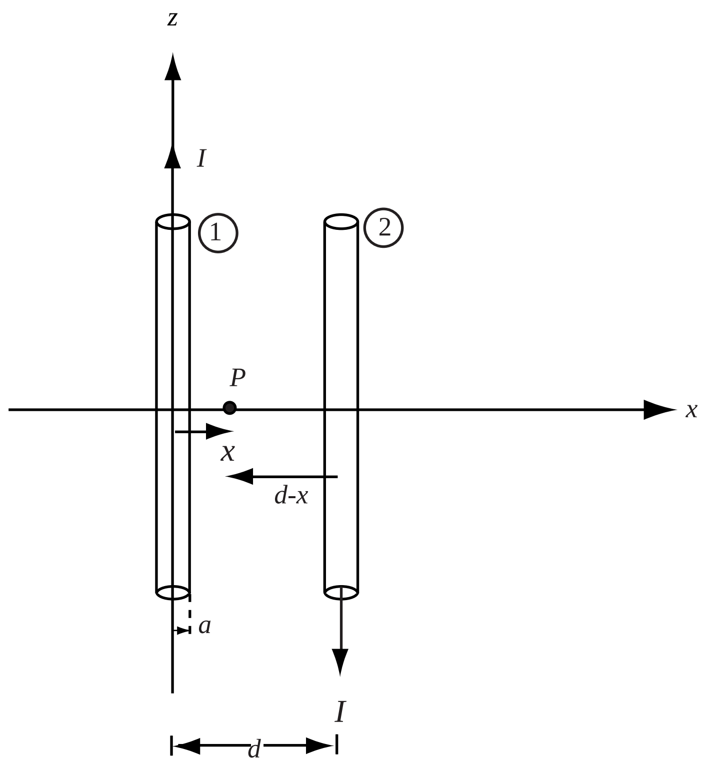

Figure P5.37: Parallel wire transmission line.

Let us place the two wires in the x–z plane and orient the current in one of them to be along the $+z$-direction and the current in the other one to be along the $-z$-direction, as shown in Fig. P5.37. From Eq. (5.30), the magnetic field at point $P = (x, 0, z)$ due to wire 1 is

$$\mathbf{B}_1 = \hat{\boldsymbol{\phi}} \frac{\mu I}{2\pi r} = \hat{\mathbf{y}} \frac{\mu I}{2\pi x},$$

where the permeability has been generalized from free space to any substance with permeability μ, and it has been recognized that in the x-z plane, $\hat{\boldsymbol{\phi}} = \hat{\mathbf{y}}$ and $r = x$ as long as $x > 0$.

Given that the current in wire 2 is opposite that in wire 1, the magnetic field created by wire 2 at point $P = (x, 0, z)$ is in the same direction as that created by wire 1, and it is given by

$$\mathbf{B}_2 = \hat{\mathbf{y}} \frac{\mu I}{2\pi(d - x)}.$$

Therefore, the total magnetic field in the region between the wires is

$$\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = \hat{\mathbf{y}} \frac{\mu I}{2\pi} \left(\frac{1}{x} + \frac{1}{d - x} \right) = \hat{\mathbf{y}} \frac{\mu I d}{2\pi x(d - x)}.$$

From Eq. (5.91), the flux crossing the surface area between the wires over a length l of the wire structure is

$$\begin{aligned}
\Phi = \iint_S \mathbf{B} \cdot d\mathbf{s} &= \int_{z=z_0}^{z_0+l} \int_{x=a}^{d-a} \left(\hat{\mathbf{y}} \frac{\mu I d}{2\pi x(d - x)} \right) \cdot (\hat{\mathbf{y}}\, dx\, dz) \\
&= \frac{\mu I l d}{2\pi} \left(\frac{1}{d} \ln \left(\frac{x}{d - x} \right) \right) \bigg|_{x=a}^{d-a} \\
&= \frac{\mu I l}{2\pi} \left(\ln \left(\frac{d - a}{a} \right) - \ln \left(\frac{a}{d - a} \right) \right) \\
&= \frac{\mu I l}{2\pi} \times 2 \ln \left(\frac{d - a}{a} \right) = \frac{\mu I l}{\pi} \ln \left(\frac{d - a}{a} \right).
\end{aligned}$$

Since the number of 'turns' in this structure is 1, Eq. (5.93) states that the flux linkage is the same as magnetic flux: $\Lambda = \Phi$. Then Eq. (5.94) gives a total inductance over the length l as

$$L = \frac{\Lambda}{I} = \frac{\Phi}{I} = \frac{\mu l}{\pi} \ln \left(\frac{d - a}{a} \right) \quad \text{(H)}.$$

Therefore, the inductance per unit length is

$$L' = \frac{L}{l} = \frac{\mu}{\pi} \ln \left(\frac{d - a}{a} \right) \approx \frac{\mu}{\pi} \ln \left(\frac{d}{a} \right) \quad \text{(H/m)},$$

where the last approximation recognizes that the wires are thin compared to the separation distance (i.e., that $d \gg a$). This has been an implied condition from the beginning of this analysis, where the flux passing through the wires themselves have been ignored. This is the thin-wire limit in Table 2-1 for the two wire line.