

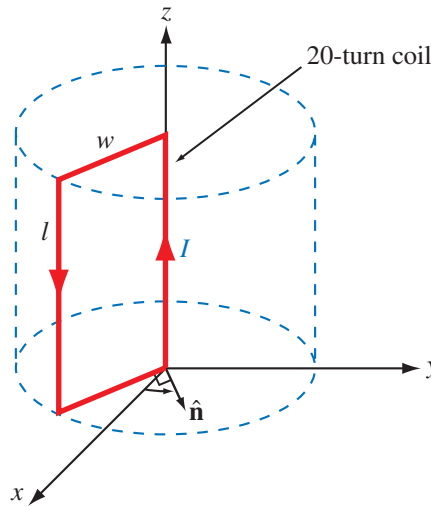
**Problem 5.6** A 20-turn rectangular coil with sides  $l = 30$  cm and  $w = 10$  cm is placed in the  $y$ - $z$  plane as shown in Fig. P5.6.

- (a) If the coil, which carries a current  $I = 10$  A, is in the presence of a magnetic flux density

$$\mathbf{B} = 2 \times 10^{-2}(\hat{\mathbf{x}} + \hat{\mathbf{y}}2) \quad (\text{T}),$$

determine the torque acting on the coil.

- (b) At what angle  $\phi$  is the torque zero?  
(c) At what angle  $\phi$  is the torque maximum? Determine its value.



**Figure P5.6:** Rectangular loop of Problem 5.6.

**Solution:**

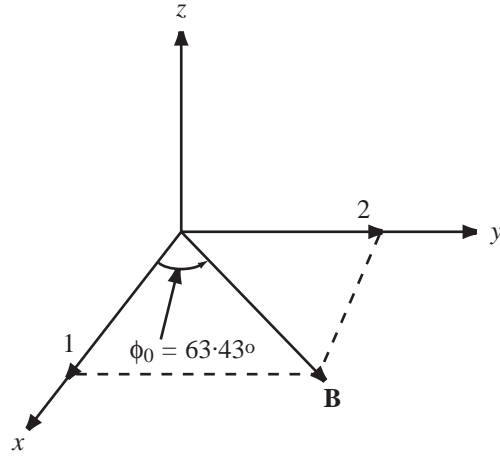
- (a) The magnetic field is in direction  $(\hat{\mathbf{x}} + \hat{\mathbf{y}}2)$ , which makes an angle  $\phi_0 = \tan^{-1} \frac{2}{1} = 63.43^\circ$ .

The magnetic moment of the loop is

$$\mathbf{m} = \hat{\mathbf{n}}NIA = \hat{\mathbf{n}}20 \times 10 \times (30 \times 10) \times 10^{-4} = \hat{\mathbf{n}}6 \quad (\text{A}\cdot\text{m}^2),$$

where  $\hat{\mathbf{n}}$  is the surface normal in accordance with the right-hand rule. When the loop is in the negative- $y$  of the  $y$ - $z$  plane,  $\hat{\mathbf{n}}$  is equal to  $\hat{\mathbf{x}}$ , but when the plane of the loop is moved to an angle  $\phi$ ,  $\hat{\mathbf{n}}$  becomes

$$\hat{\mathbf{n}} = \hat{\mathbf{x}} \cos \phi + \hat{\mathbf{y}} \sin \phi,$$



**Figure P5.6:** (a) Direction of  $\mathbf{B}$ .

$$\begin{aligned}\mathbf{T} &= \mathbf{m} \times \mathbf{B} = \hat{\mathbf{n}} 6 \times 2 \times 10^{-2} (\hat{\mathbf{x}} + \hat{\mathbf{y}} 2) \\ &= (\hat{\mathbf{x}} \cos \phi + \hat{\mathbf{y}} \sin \phi) 6 \times 2 \times 10^{-2} (\hat{\mathbf{x}} + \hat{\mathbf{y}} 2) \\ &= \hat{\mathbf{z}} 0.12 [2 \cos \phi - \sin \phi] \quad (\text{N} \cdot \text{m}).\end{aligned}$$

**(b)** The torque is zero when

$$2 \cos \phi - \sin \phi = 0,$$

or

$$\tan \phi = 2, \quad \phi = 63.43^\circ \text{ or } -116.57^\circ.$$

Thus, when  $\hat{\mathbf{n}}$  is parallel to  $\mathbf{B}$ ,  $\mathbf{T} = 0$ .

**(c)** The torque is a maximum when  $\hat{\mathbf{n}}$  is perpendicular to  $\mathbf{B}$ , which occurs at

$$\phi = 63.43 \pm 90^\circ = -26.57^\circ \text{ or } +153.43^\circ.$$

Mathematically, we can obtain the same result by taking the derivative of  $\mathbf{T}$  and equating it to zero to find the values of  $\phi$  at which  $|\mathbf{T}|$  is a maximum. Thus,

$$\frac{\partial T}{\partial \phi} = \frac{\partial}{\partial \phi} (0.12(2 \cos \phi - \sin \phi)) = 0$$

or

$$-2 \sin \phi + \cos \phi = 0,$$

which gives  $\tan \phi = -\frac{1}{2}$ , or

$$\phi = -26.57^\circ \text{ or } 153.43^\circ,$$

at which  $\mathbf{T} = \hat{\mathbf{z}}0.27 \text{ (N}\cdot\text{m)}$ .

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