

Problem 5.7 An $8\text{ cm} \times 12\text{ cm}$ rectangular loop of wire is situated in the x - y plane with the center of the loop at the origin and its long sides parallel to the x -axis. The loop has a current of 50 A flowing clockwise (when viewed from above). Determine the magnetic field at the center of the loop.

Solution: The total magnetic field is the vector sum of the individual fields of each of the four wire segments: $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 + \mathbf{B}_3 + \mathbf{B}_4$. An expression for the magnetic field from a wire segment is given by Eq. (5.29). For all segments shown in Fig. P5.7, the

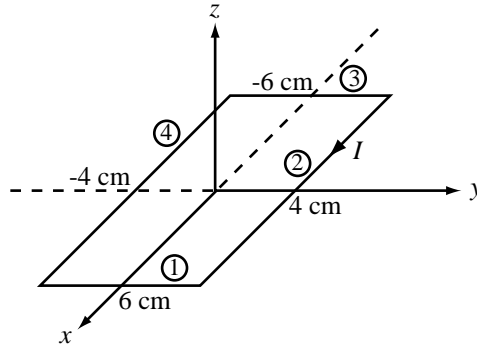


Figure P5.7: Problem 5.7.

combination of the direction of the current and the right-hand rule gives the direction of the magnetic field as $-z$ direction at the origin. With $r = 6\text{ cm}$ and $l = 8\text{ cm}$,

$$\begin{aligned}\mathbf{B}_1 &= -\hat{\mathbf{z}} \frac{\mu I l}{2\pi r \sqrt{4r^2 + l^2}} \\ &= -\hat{\mathbf{z}} \frac{4\pi \times 10^{-7} \times 50 \times 0.08}{2\pi \times 0.06 \times \sqrt{4 \times 0.06^2 + 0.08^2}} = -\hat{\mathbf{z}} 9.24 \times 10^{-5} \quad (\text{T}).\end{aligned}$$

For segment 2, $r = 4\text{ cm}$ and $l = 12\text{ cm}$,

$$\begin{aligned}\mathbf{B}_2 &= -\hat{\mathbf{z}} \frac{\mu I l}{2\pi r \sqrt{4r^2 + l^2}} \\ &= -\hat{\mathbf{z}} \frac{4\pi \times 10^{-7} \times 50 \times 0.12}{2\pi \times 0.04 \times \sqrt{4 \times 0.04^2 + 0.12^2}} = -\hat{\mathbf{z}} 20.80 \times 10^{-5} \quad (\text{T}).\end{aligned}$$

Similarly,

$$\mathbf{B}_3 = -\hat{\mathbf{z}} 9.24 \times 10^{-5} \quad (\text{T}), \quad \mathbf{B}_4 = -\hat{\mathbf{z}} 20.80 \times 10^{-5} \quad (\text{T}).$$

The total field is then $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 + \mathbf{B}_3 + \mathbf{B}_4 = -\hat{\mathbf{z}} 0.60 \text{ (mT)}$.