Problem 5.8 Use the approach outlined in Example 5-2 to develop an expression for the magnetic field **H** at an arbitrary point P due to the linear conductor defined by the geometry shown in Fig. P5.8. If the conductor extends between $z_1 = 3$ m and $z_2 = 7$ m and carries a current $I = 15$ A, find **H** at $P = (2, \phi, 0)$.

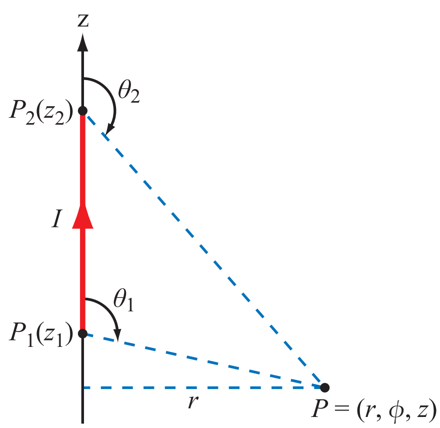

Figure P5.8: Current-carrying linear conductor of Problem 5.8.

Solution: The solution follows Example 5-2 up through Eq. (5.27), but the expressions for the cosines of the angles should be generalized to read as

$$\cos\theta_1 = \frac{z - z_1}{\sqrt{r^2 + (z - z_1)^2}}, \qquad \cos\theta_2 = \frac{z - z_2}{\sqrt{r^2 + (z - z_2)^2}}$$

instead of the expressions in Eq. (5.28), which are specialized to a wire centered at the origin. Plugging these expressions back into Eq. (5.27), the magnetic field is given as

$$\mathbf{H} = \hat{\boldsymbol{\phi}} \frac{I}{4\pi r} \left(\frac{z - z_1}{\sqrt{r^2 + (z - z_1)^2}} - \frac{z - z_2}{\sqrt{r^2 + (z - z_2)^2}} \right).$$

For the specific geometry of Fig. ,

$$\mathbf{H} = \hat{\boldsymbol{\phi}} \frac{15}{4\pi \times 2} \left[\frac{0 - 3}{\sqrt{3^2 + 2^2}} - \frac{0 - 7}{\sqrt{7^2 + 2^2}} \right] = \hat{\boldsymbol{\phi}}\, 77.4 \times 10^{-3} \text{ (A/m)} = \hat{\boldsymbol{\phi}}\, 77.4 \quad \text{(mA/m)}.$$