Problem 6.11 The loop shown in P6.11 moves away from a wire carrying a current $I_1 = 10$ A at a constant velocity $\mathbf{u} = \hat{\mathbf{y}}7.5$ (m/s). If $R = 10\ \Omega$ and the direction of I_2 is as defined in the figure, find I_2 as a function of y_0, the distance between the wire and the loop. Ignore the internal resistance of the loop.

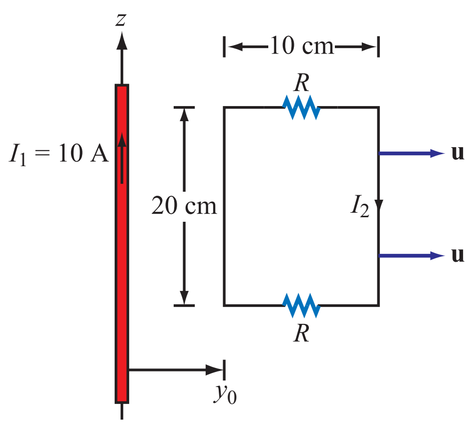

Figure P6.11: Moving loop of Problem 6.11.

Solution: Assume that the wire carrying current I_1 is in the same plane as the loop. The two identical resistors are in series, so $I_2 = V_{\text{emf}}/2R$, where the induced voltage is due to motion of the loop and is given by Eq. (6.26):

$$V_{\text{emf}} = V_{\text{emf}}^{\text{m}} = \oint_C (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}.$$

The magnetic field $\mathbf{B}$ is created by the wire carrying I_1. Choosing $\hat{\mathbf{z}}$ to coincide with the direction of I_1, Eq. (5.30) gives the external magnetic field of a long wire to be

$$\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I_1}{2\pi r} .$$

For positive values of y_0 in the y-z plane, $\hat{\mathbf{y}} = \hat{\mathbf{r}}$, so

$$\mathbf{u} \times \mathbf{B} = \hat{\mathbf{y}}|\mathbf{u}| \times \mathbf{B} = \hat{\mathbf{r}}|\mathbf{u}| \times \hat{\boldsymbol{\phi}} \frac{\mu_0 I_1}{2\pi r} = \hat{\mathbf{z}} \frac{\mu_0 I_1 u}{2\pi r} .$$

Integrating around the four sides of the loop with $d\mathbf{l} = \hat{\mathbf{z}}\, dz$ and the limits of integration chosen in accordance with the assumed direction of I_2, and recognizing that only the two sides without the resistors contribute to $V^{\mathrm{m}}_{\mathrm{emf}}$, we have

$$\begin{aligned}
V^{\mathrm{m}}_{\mathrm{emf}} &= \int_0^{0.2} \left(\hat{\mathbf{z}}\frac{\mu_0 I_1 u}{2\pi r}\right)\bigg|_{r=y_0} \cdot (\hat{\mathbf{z}}\, dz) + \int_{0.2}^{0} \left(\hat{\mathbf{z}}\frac{\mu_0 I_1 u}{2\pi r}\right)\bigg|_{r=y_0+0.1} \cdot (\hat{\mathbf{z}}\, dz) \\
&= \frac{4\pi \times 10^{-7} \times 10 \times 7.5 \times 0.2}{2\pi} \left(\frac{1}{y_0} - \frac{1}{y_0+0.1}\right) \\
&= 3 \times 10^{-6} \left(\frac{1}{y_0} - \frac{1}{y_0+0.1}\right) \quad \text{(V)},
\end{aligned}$$

and therefore

$$I_2 = \frac{V^{\mathrm{m}}_{\mathrm{emf}}}{2R} = 150 \left(\frac{1}{y_0} - \frac{1}{y_0+0.1}\right) \quad \text{(nA)}.$$