Problem 6.15 A coaxial capacitor of length $l = 6$ cm uses an insulating dielectric material with $\varepsilon_r = 9$. The radii of the cylindrical conductors are 0.5 cm and 1 cm. If the voltage applied across the capacitor is

$$V(t) = 50\sin(120\pi t) \quad \text{(V)}$$

what is the displacement current?

Solution:

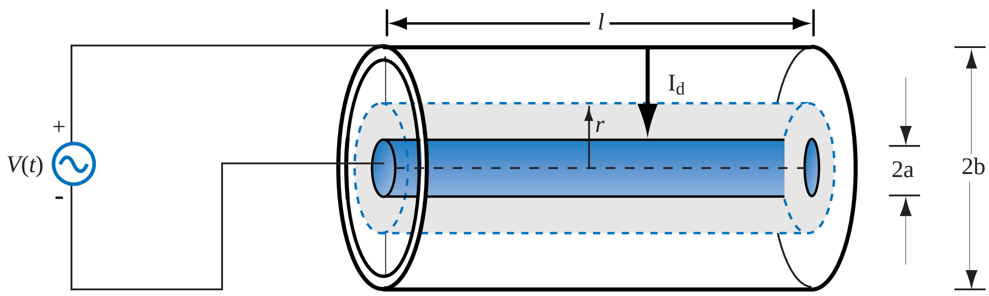

Figure P6.15:

To find the displacement current, we need to know **E** in the dielectric space between the cylindrical conductors. From Eqs. (4.114) and (4.115),

$$\mathbf{E} = -\hat{\mathbf{r}}\frac{Q}{2\pi\varepsilon rl},$$

$$V = \frac{Q}{2\pi\varepsilon l}\ln\left(\frac{b}{a}\right).$$

Hence,

$$\mathbf{E} = -\hat{\mathbf{r}}\frac{V}{r\ln\left(\frac{b}{a}\right)} = -\hat{\mathbf{r}}\frac{50\sin(120\pi t)}{r\ln 2} = -\hat{\mathbf{r}}\frac{72.1}{r}\sin(120\pi t) \quad \text{(V/m)},$$

$$\begin{aligned}\mathbf{D} &= \varepsilon\mathbf{E}\\ &= \varepsilon_r\varepsilon_0\mathbf{E}\\ &= -\hat{\mathbf{r}}\,9\times 8.85\times 10^{-12}\times\frac{72.1}{r}\sin(120\pi t)\\ &= -\hat{\mathbf{r}}\frac{5.75\times 10^{-9}}{r}\sin(120\pi t) \quad \text{(C/m}^2\text{)}.\end{aligned}$$

The displacement current flows between the conductors through an imaginary cylindrical surface of length l and radius r. The current flowing from the outer conductor to the inner conductor along $-\hat{\mathbf{r}}$ crosses surface **S** where

$$\mathbf{S} = -\hat{\mathbf{r}}\,2\pi rl.$$

Hence,

$$I_{\rm d} = \frac{\partial \mathbf{D}}{\partial t} \cdot \mathbf{S} = -\hat{\mathbf{r}} \frac{\partial}{\partial t} \left(\frac{5.75 \times 10^{-9}}{r} \sin(120\pi t) \right) \cdot (-\hat{\mathbf{r}} 2\pi r l)$$
$$= 5.75 \times 10^{-9} \times 120\pi \times 2\pi l \cos(120\pi t)$$
$$= 0.82 \cos(120\pi t) \quad (\mu\text{A}).$$

Alternatively, since the coaxial capacitor is lossless, its displacement current has to be equal to the conduction current flowing through the wires connected to the voltage sources. The capacitance of a coaxial capacitor is given by (4.116) as

$$C = \frac{2\pi\varepsilon l}{\ln\left(\frac{b}{a}\right)}.$$

The current is

$$I = C \frac{dV}{dt} = \frac{2\pi\varepsilon l}{\ln\left(\frac{b}{a}\right)} [120\pi \times 50 \cos(120\pi t)] = 0.82 \cos(120\pi t) \quad (\mu\text{A}),$$

which is the same answer we obtained before.