

Problem 6.16 The parallel-plate capacitor shown in Fig. P6.16 is filled with a lossy dielectric material of relative permittivity ϵ_r and conductivity σ . The separation between the plates is d and each plate is of area A . The capacitor is connected to a time-varying voltage source $V(t)$.

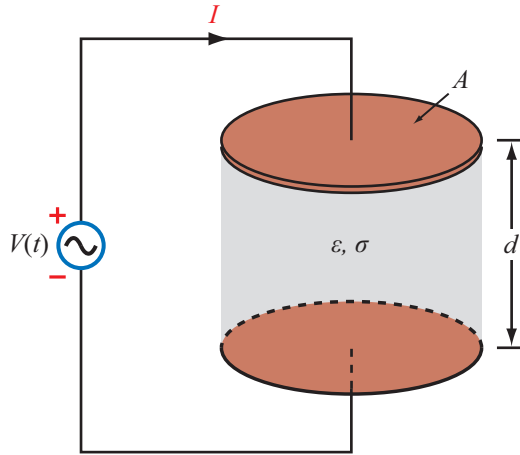


Figure P6.16: Parallel-plate capacitor containing a lossy dielectric material (Problem 6.16).

- (a) Obtain an expression for I_c , the conduction current flowing between the plates inside the capacitor, in terms of the given quantities.
- (b) Obtain an expression for I_d , the displacement current flowing inside the capacitor.
- (c) Based on your expressions for parts (a) and (b), give an equivalent-circuit representation for the capacitor.
- (d) Evaluate the values of the circuit elements for $A = 4 \text{ cm}^2$, $d = 0.5 \text{ cm}$, $\epsilon_r = 4$, $\sigma = 2.5 \text{ (S/m)}$, and $V(t) = 10 \cos(3\pi \times 10^3 t) \text{ (V)}$.

Solution:

(a)

$$R = \frac{d}{\sigma A}, \quad I_c = \frac{V}{R} = \frac{V \sigma A}{d}.$$

(b)

$$E = \frac{V}{d}, \quad I_d = \frac{\partial D}{\partial t} \cdot A = \epsilon A \frac{\partial E}{\partial t} = \frac{\epsilon A}{d} \frac{\partial V}{\partial t}.$$

(c) The conduction current is directly proportional to V , as characteristic of a resistor, whereas the displacement current varies as $\partial V / \partial t$, which is characteristic

of a capacitor. Hence,

$$R = \frac{d}{\sigma A} \quad \text{and} \quad C = \frac{\epsilon A}{d}.$$

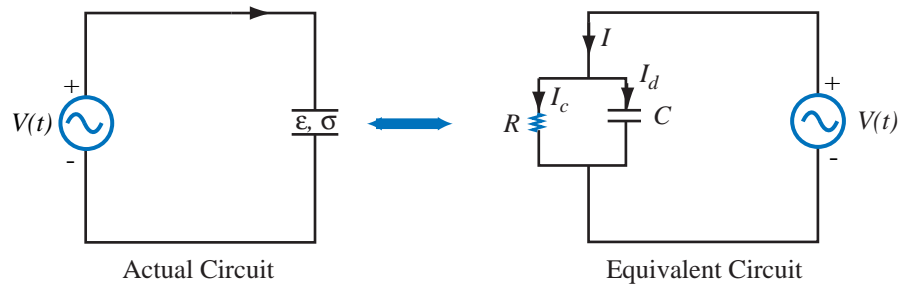


Figure P6.16: (a) Equivalent circuit.

(d)

$$R = \frac{0.5 \times 10^{-2}}{2.5 \times 4 \times 10^{-4}} = 5 \, \Omega,$$

$$C = \frac{4 \times 8.85 \times 10^{-12} \times 4 \times 10^{-4}}{0.5 \times 10^{-2}} = 2.84 \times 10^{-12} \, \text{F}.$$
