

Problem 6.17 In wet soil, characterized by $\sigma = 10^{-2}$ (S/m), $\mu_r = 1$, and $\epsilon_r = 36$, at what frequency is the conduction current density equal in magnitude to the displacement current density?

Solution: For sinusoidal wave variation, the phasor electric field is

$$\begin{aligned}E &= E_0 e^{j\omega t} \\J_c &= \sigma E = \sigma E_0 e^{j\omega t} \\J_d &= \frac{\partial D}{\partial t} = \epsilon \frac{\partial E}{\partial t} = j\omega \epsilon E_0 e^{j\omega t} \\\left| \frac{J_c}{J_d} \right| &= 1 = \frac{\sigma}{\omega \epsilon} = \frac{\sigma}{2\pi \epsilon f}\end{aligned}$$

or

$$f = \frac{\sigma}{2\pi \epsilon} = \frac{10^{-2}}{2\pi \times 36 \times 8.85 \times 10^{-12}} = 5 \times 10^6 = 5 \text{ MHz.}$$
