

Problem 6.18 An electromagnetic wave propagating in seawater has an electric field with a time variation given by $\mathbf{E} = \hat{\mathbf{z}}E_0 \cos \omega t$. If the permittivity of water is $81\epsilon_0$ and its conductivity is 4 (S/m) , find the ratio of the magnitudes of the conduction current density to displacement current density at each of the following frequencies:

- (a) 1 kHz
- (b) 1 MHz
- (c) 1 GHz
- (d) 100 GHz

Solution: From Eq. (6.44), the displacement current density is given by

$$\mathbf{J}_d = \frac{\partial}{\partial t} \mathbf{D} = \epsilon \frac{\partial}{\partial t} \mathbf{E}$$

and, from Eq. (4.67), the conduction current is $\mathbf{J} = \sigma \mathbf{E}$. Converting to phasors and taking the ratio of the magnitudes,

$$\left| \frac{\tilde{\mathbf{J}}}{\tilde{\mathbf{J}}_d} \right| = \left| \frac{\sigma \tilde{\mathbf{E}}}{j\omega \epsilon_r \epsilon_0 \tilde{\mathbf{E}}} \right| = \frac{\sigma}{\omega \epsilon_r \epsilon_0}.$$

- (a) At $f = 1 \text{ kHz}$, $\omega = 2\pi \times 10^3 \text{ rad/s}$, and

$$\left| \frac{\tilde{\mathbf{J}}}{\tilde{\mathbf{J}}_d} \right| = \frac{4}{2\pi \times 10^3 \times 81 \times 8.854 \times 10^{-12}} = 888 \times 10^3.$$

The displacement current is negligible.

- (b) At $f = 1 \text{ MHz}$, $\omega = 2\pi \times 10^6 \text{ rad/s}$, and

$$\left| \frac{\tilde{\mathbf{J}}}{\tilde{\mathbf{J}}_d} \right| = \frac{4}{2\pi \times 10^6 \times 81 \times 8.854 \times 10^{-12}} = 888.$$

The displacement current is practically negligible.

- (c) At $f = 1 \text{ GHz}$, $\omega = 2\pi \times 10^9 \text{ rad/s}$, and

$$\left| \frac{\tilde{\mathbf{J}}}{\tilde{\mathbf{J}}_d} \right| = \frac{4}{2\pi \times 10^9 \times 81 \times 8.854 \times 10^{-12}} = 0.888.$$

Neither the displacement current nor the conduction current are negligible.

- (d) At $f = 100 \text{ GHz}$, $\omega = 2\pi \times 10^{11} \text{ rad/s}$, and

$$\left| \frac{\tilde{\mathbf{J}}}{\tilde{\mathbf{J}}_d} \right| = \frac{4}{2\pi \times 10^{11} \times 81 \times 8.854 \times 10^{-12}} = 8.88 \times 10^{-3}.$$

The conduction current is practically negligible.
