Problem 6.2 The loop in Fig. P6.2 is in the x–y plane and $\mathbf{B} = \hat{\mathbf{z}} B_0 \sin \omega t$ with B_0 positive. What is the direction of I ($\hat{\boldsymbol{\phi}}$ or $-\hat{\boldsymbol{\phi}}$) at:

(a) $t = 0$

(b) $\omega t = \pi/4$

(c) $\omega t = \pi/2$

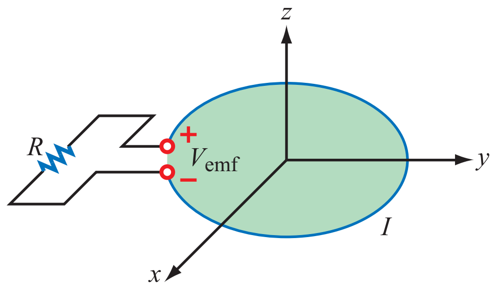

Figure P6.2: Loop of Problem 6.2.

Solution: $I = V_{\text{emf}}/R$. Since the single-turn loop is not moving or changing shape with time, $V_{\text{emf}}^{\text{m}} = 0$ V and $V_{\text{emf}} = V_{\text{emf}}^{\text{tr}}$. Therefore, from Eq. (6.8),

$$I = V_{\text{emf}}^{\text{tr}}/R = \frac{-1}{R} \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}.$$

If we take the surface normal to be $+\hat{\mathbf{z}}$, then the right hand rule gives positive flowing current to be in the $+\hat{\boldsymbol{\phi}}$ direction.

$$I = \frac{-A}{R} \frac{\partial}{\partial t} B_0 \sin \omega t = \frac{-A B_0 \omega}{R} \cos \omega t \quad \text{(A)},$$

where A is the area of the loop.

(a) A, ω and R are positive quantities. At $t = 0$, $\cos \omega t = 1$ so $I < 0$ and the current is flowing in the $-\hat{\boldsymbol{\phi}}$ direction (so as to produce an induced magnetic field that opposes $\mathbf{B}$).

(b) At $\omega t = \pi/4$, $\cos \omega t = \sqrt{2}/2$ so $I < 0$ and the current is still flowing in the $-\hat{\boldsymbol{\phi}}$ direction.

(c) At $\omega t = \pi/2$, $\cos \omega t = 0$ so $I = 0$. There is no current flowing in either direction.