

Problem 6.20 If the current density in a conducting medium is given by

$$\mathbf{J}(x, y, z; t) = (\hat{\mathbf{x}}z - \hat{\mathbf{y}}3y^2 + \hat{\mathbf{z}}2x) \cos \omega t$$

determine the corresponding charge distribution $\rho_v(x, y, z; t)$.

Solution: Eq. (6.58) is given by

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t}. \quad (28)$$

The divergence of \mathbf{J} is

$$\begin{aligned} \nabla \cdot \mathbf{J} &= \left(\hat{\mathbf{x}} \frac{\partial}{\partial x} + \hat{\mathbf{y}} \frac{\partial}{\partial y} + \hat{\mathbf{z}} \frac{\partial}{\partial z} \right) \cdot (\hat{\mathbf{x}}z - \hat{\mathbf{y}}4y^2 + \hat{\mathbf{z}}2x) \cos \omega t \\ &= -4 \frac{\partial}{\partial y} (y^2 \cos \omega t) = -8y \cos \omega t. \end{aligned}$$

Using this result in Eq. (28) and then integrating both sides with respect to t gives

$$\rho_v = - \int (\nabla \cdot \mathbf{J}) dt = - \int -8y \cos \omega t dt = \frac{8y}{\omega} \sin \omega t + C_0,$$

where C_0 is a constant of integration.
