

Problem 6.23 The electric field of an electromagnetic wave propagating in air is given by

$$\mathbf{E}(z, t) = \hat{\mathbf{x}}4 \cos(6 \times 10^8 t - 2z) + \hat{\mathbf{y}}3 \sin(6 \times 10^8 t - 2z) \quad (\text{V/m}).$$

Find the associated magnetic field $\mathbf{H}(z, t)$.

Solution: Converting to phasor form, the electric field is given by

$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}4e^{-j2z} - j\hat{\mathbf{y}}3e^{-j2z} \quad (\text{V/m}),$$

which can be used with Eq. (6.87) to find the magnetic field:

$$\begin{aligned} \tilde{\mathbf{H}}(z) &= \frac{1}{-j\omega\mu} \nabla \times \tilde{\mathbf{E}} \\ &= \frac{1}{-j\omega\mu} \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 4e^{-j2z} & -j3e^{-j2z} & 0 \end{vmatrix} \\ &= \frac{1}{-j\omega\mu} (\hat{\mathbf{x}}6e^{-j2z} - \hat{\mathbf{y}}j8e^{-j2z}) \\ &= \frac{j}{6 \times 10^8 \times 4\pi \times 10^{-7}} (\hat{\mathbf{x}}6 - \hat{\mathbf{y}}j8)e^{-j2z} = j\hat{\mathbf{x}}8.0e^{-j2z} + \hat{\mathbf{y}}10.6e^{-j2z} \quad (\text{mA/m}). \end{aligned}$$

Converting back to instantaneous values, this is

$$\mathbf{H}(t, z) = -\hat{\mathbf{x}}8.0 \sin(6 \times 10^8 t - 2z) + \hat{\mathbf{y}}10.6 \cos(6 \times 10^8 t - 2z) \quad (\text{mA/m}).$$
