

Problem 6.25 Given an electric field

$$\mathbf{E} = \hat{\mathbf{x}}E_0 \sin ay \cos(\omega t - kz),$$

where E_0 , a , ω , and k are constants, find \mathbf{H} .

Solution:

$$\mathbf{E} = \hat{\mathbf{x}}E_0 \sin ay \cos(\omega t - kz),$$

$$\tilde{\mathbf{E}} = \hat{\mathbf{x}}E_0 \sin ay e^{-jkz},$$

$$\tilde{\mathbf{H}} = -\frac{1}{j\omega\mu} \nabla \times \tilde{\mathbf{E}}$$

$$= -\frac{1}{j\omega\mu} \left[\hat{\mathbf{y}} \frac{\partial}{\partial z} (E_0 \sin ay e^{-jkz}) - \hat{\mathbf{z}} \frac{\partial}{\partial y} (E_0 \sin ay e^{-jkz}) \right]$$

$$= \frac{E_0}{\omega\mu} [\hat{\mathbf{y}} k \sin ay - \hat{\mathbf{z}} j a \cos ay] e^{-jkz},$$

$$\mathbf{H} = \Re[\tilde{\mathbf{H}} e^{j\omega t}]$$

$$= \Re \left\{ \frac{E_0}{\omega\mu} [\hat{\mathbf{y}} k \sin ay + \hat{\mathbf{z}} a \cos ay e^{-j\pi/2}] e^{-jkz} e^{j\omega t} \right\}$$

$$= \frac{E_0}{\omega\mu} \left[\hat{\mathbf{y}} k \sin ay \cos(\omega t - kz) + \hat{\mathbf{z}} a \cos ay \cos\left(\omega t - kz - \frac{\pi}{2}\right) \right]$$

$$= \frac{E_0}{\omega\mu} [\hat{\mathbf{y}} k \sin ay \cos(\omega t - kz) + \hat{\mathbf{z}} a \cos ay \sin(\omega t - kz)].$$
