

Problem 6.26 The electric field radiated by a short dipole antenna is given in spherical coordinates by

$$\mathbf{E}(R, \theta; t) = \hat{\boldsymbol{\theta}} \frac{2 \times 10^{-2}}{R} \sin \theta \cos(6\pi \times 10^8 t - 2\pi R) \quad (\text{V/m}).$$

Find $\mathbf{H}(R, \theta; t)$.

Solution: Converting to phasor form, the electric field is given by

$$\tilde{\mathbf{E}}(R, \theta) = \hat{\boldsymbol{\theta}} E_{\theta} = \hat{\boldsymbol{\theta}} \frac{2 \times 10^{-2}}{R} \sin \theta e^{-j2\pi R} \quad (\text{V/m}),$$

which can be used with Eq. (6.87) to find the magnetic field:

$$\begin{aligned} \tilde{\mathbf{H}}(R, \theta) &= \frac{1}{-j\omega\mu} \nabla \times \tilde{\mathbf{E}} = \frac{1}{-j\omega\mu} \left[\hat{\mathbf{R}} \frac{1}{R \sin \theta} \frac{\partial E_{\theta}}{\partial \phi} + \hat{\boldsymbol{\phi}} \frac{1}{R} \frac{\partial}{\partial R} (R E_{\theta}) \right] \\ &= \frac{1}{-j\omega\mu} \hat{\boldsymbol{\phi}} \frac{2 \times 10^{-2}}{R} \sin \theta \frac{\partial}{\partial R} (e^{-j2\pi R}) \\ &= \hat{\boldsymbol{\phi}} \frac{2\pi}{6\pi \times 10^8 \times 4\pi \times 10^{-7}} \frac{2 \times 10^{-2}}{R} \sin \theta e^{-j2\pi R} \\ &= \hat{\boldsymbol{\phi}} \frac{53}{R} \sin \theta e^{-j2\pi R} \quad (\mu\text{A/m}). \end{aligned}$$

Converting back to instantaneous value, this is

$$\mathbf{H}(R, \theta; t) = \hat{\boldsymbol{\phi}} \frac{53}{R} \sin \theta \cos(6\pi \times 10^8 t - 2\pi R) \quad (\mu\text{A/m}).$$
