Problem 6.28 In free space, the magnetic field is given by

$$\mathbf{H} = \hat{\boldsymbol{\phi}} \frac{36}{r} \cos(6 \times 10^9 t - kz) \quad \text{(mA/m)}.$$

(a) Determine k.

(b) Determine $\mathbf{E}$.

(c) Determine $\mathbf{J}_\text{d}$.

Solution:

(a) From the given expression, $\omega = 6 \times 10^9$ (rad/s), and since the medium is free space,

$$k = \frac{\omega}{c} = \frac{6 \times 10^9}{3 \times 10^8} = 20 \quad \text{(rad/m)}.$$

(b) Convert $\mathbf{H}$ to phasor:

$$\begin{aligned}
\widetilde{\mathbf{H}} &= \hat{\boldsymbol{\phi}} \frac{36}{r} e^{-jkz} \quad \text{(mA/m)} \\
\widetilde{\mathbf{E}} &= \frac{1}{j\omega\varepsilon_0} \nabla \times \widetilde{\mathbf{H}} \\
&= \frac{1}{j\omega\varepsilon_0} \left[-\hat{\mathbf{r}} \frac{\partial H_\phi}{\partial z} + \hat{\mathbf{z}} \frac{1}{r} \frac{\partial}{\partial r}(rH_\phi) \right] \\
&= \frac{1}{j\omega\varepsilon_0} \left[-\hat{\mathbf{r}} \frac{\partial}{\partial z} \left(\frac{36}{r} e^{-jkz} \right) + \frac{\hat{\mathbf{z}}}{r} \frac{\partial}{\partial r}(36e^{-jkz}) \right] \\
&= \frac{1}{j\omega\varepsilon_0} \left[\hat{\mathbf{r}} \frac{j36k}{r} e^{-jkz} \right] \\
&= \hat{\mathbf{r}} \frac{36k}{\omega\varepsilon_0 r} e^{-jkz} = \hat{\mathbf{r}} \frac{36 \times 377}{r} e^{-jkz} \times 10^{-3} = \hat{\mathbf{r}} \frac{13.6}{r} e^{-j20z} \quad \text{(V/m)}. \\
\mathbf{E} &= \mathfrak{Re}[\widetilde{E} e^{j\omega t}] \\
&= \hat{\mathbf{r}} \frac{13.6}{r} \cos(6 \times 10^9 t - 20z) \quad \text{(V/m)}.
\end{aligned}$$

(c)

$$\begin{aligned}
\mathbf{J}_\text{d} &= \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \\
&= \hat{\mathbf{r}} \frac{13.6}{r} \varepsilon_0 \frac{\partial}{\partial t} (\cos(6 \times 10^9 t - 20z)) \\
&= -\hat{\mathbf{r}} \frac{13.6\varepsilon_0 \times 6 \times 10^9}{r} \sin(6 \times 10^9 t - 20z) \quad \text{(A/m}^2\text{)} \\
&= -\hat{\mathbf{r}} \frac{0.72}{r} \sin(6 \times 10^9 t - 20z) \quad \text{(A/m}^2\text{)}.
\end{aligned}$$