

**Problem 6.3** A coil consists of 100 turns of wire wrapped around a square frame of sides 0.25 m. The coil is centered at the origin with each of its sides parallel to the  $x$ - or  $y$ -axis. Find the induced emf across the open-circuited ends of the coil if the magnetic field is given by

(a)  $\mathbf{B} = \hat{\mathbf{z}} 20e^{-3t}$  (T)

(b)  $\mathbf{B} = \hat{\mathbf{z}} 20 \cos x \cos 10^3 t$  (T)

(c)  $\mathbf{B} = \hat{\mathbf{z}} 20 \cos x \sin 2y \cos 10^3 t$  (T)

**Solution:** Since the coil is not moving or changing shape,  $V_{\text{emf}}^{\text{m}} = 0$  V and  $V_{\text{emf}} = V_{\text{emf}}^{\text{tr}}$ . From Eq. (6.6),

$$V_{\text{emf}} = -N \frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{s} = -N \frac{d}{dt} \int_{-0.125}^{0.125} \int_{-0.125}^{0.125} \mathbf{B} \cdot (\hat{\mathbf{z}} dx dy),$$

where  $N = 100$  and the surface normal was chosen to be in the  $+\hat{\mathbf{z}}$  direction.

(a) For  $\mathbf{B} = \hat{\mathbf{z}} 20e^{-3t}$  (T),

$$V_{\text{emf}} = -100 \frac{d}{dt} (20e^{-3t} (0.25)^2) = 375e^{-3t} \quad (\text{V}).$$

(b) For  $\mathbf{B} = \hat{\mathbf{z}} 20 \cos x \cos 10^3 t$  (T),

$$V_{\text{emf}} = -100 \frac{d}{dt} \left( 20 \cos 10^3 t \int_{x=-0.125}^{0.125} \int_{y=-0.125}^{0.125} \cos x dx dy \right) = 124.6 \sin 10^3 t \quad (\text{kV}).$$

(c) For  $\mathbf{B} = \hat{\mathbf{z}} 20 \cos x \sin 2y \cos 10^3 t$  (T),

$$V_{\text{emf}} = -100 \frac{d}{dt} \left( 20 \cos 10^3 t \int_{x=-0.125}^{0.125} \int_{y=-0.125}^{0.125} \cos x \sin 2y dx dy \right) = 0.$$


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