

Problem 6.6 The square loop shown in Fig. P6.6 is coplanar with a long, straight wire carrying a current

$$I(t) = 5 \cos(2\pi \times 10^4 t) \quad (\text{A}).$$

- (a) Determine the emf induced across a small gap created in the loop.
- (b) Determine the direction and magnitude of the current that would flow through a $4\text{-}\Omega$ resistor connected across the gap. The loop has an internal resistance of $1\text{ }\Omega$.

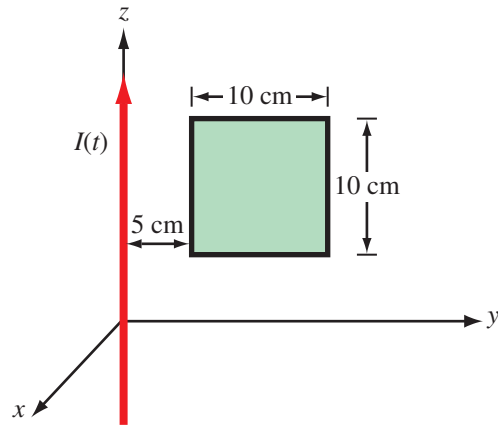


Figure P6.6: Loop coplanar with long wire (Problem 6.6).

Solution:

- (a) The magnetic field due to the wire is

$$\mathbf{B} = \hat{\phi} \frac{\mu_0 I}{2\pi r} = -\hat{\mathbf{x}} \frac{\mu_0 I}{2\pi y},$$

where in the plane of the loop, $\hat{\phi} = -\hat{\mathbf{x}}$ and $r = y$. The flux passing through the loop is

$$\begin{aligned} \Phi &= \int_S \mathbf{B} \cdot d\mathbf{s} = \int_{5\text{ cm}}^{15\text{ cm}} \left(-\hat{\mathbf{x}} \frac{\mu_0 I}{2\pi y} \right) \cdot [-\hat{\mathbf{x}} 10\text{ (cm)}] dy \\ &= \frac{\mu_0 I \times 10^{-1}}{2\pi} \ln \frac{15}{5} \\ &= \frac{4\pi \times 10^{-7} \times 5 \cos(2\pi \times 10^4 t) \times 10^{-1}}{2\pi} \times 1.1 \\ &= 1.1 \times 10^{-7} \cos(2\pi \times 10^4 t) \quad (\text{Wb}). \end{aligned}$$

$$\begin{aligned}
 V_{\text{emf}} &= -\frac{d\Phi}{dt} = 1.1 \times 2\pi \times 10^4 \sin(2\pi \times 10^4 t) \times 10^{-7} \\
 &= 6.9 \times 10^{-3} \sin(2\pi \times 10^4 t) \quad (\text{V}).
 \end{aligned}$$

(b)

$$I_{\text{ind}} = \frac{V_{\text{emf}}}{4 + 1} = \frac{6.9 \times 10^{-3}}{5} \sin(2\pi \times 10^4 t) = 1.38 \sin(2\pi \times 10^4 t) \quad (\text{mA}).$$

At $t = 0$, \mathbf{B} is a maximum, it points in $-\hat{\mathbf{x}}$ -direction, and since it varies as $\cos(2\pi \times 10^4 t)$, it is decreasing. Hence, the induced current has to be CCW when looking down on the loop, as shown in the figure.
