

Problem 6.8 The transformer shown in Fig. P6.8 consists of a long wire coincident with the z -axis carrying a current $I = I_0 \cos \omega t$, coupling magnetic energy to a toroidal coil situated in the x - y plane and centered at the origin. The toroidal core uses iron material with relative permeability μ_r , around which 100 turns of a tightly wound coil serves to induce a voltage V_{emf} , as shown in the figure.

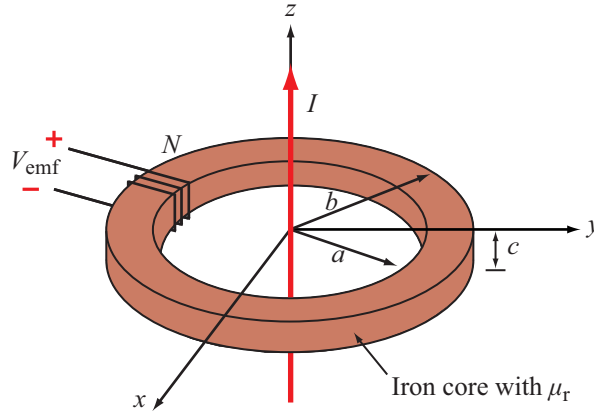


Figure P6.8: Problem 6.8.

- (a) Develop an expression for V_{emf} .
- (b) Calculate V_{emf} for $f = 60$ Hz, $\mu_r = 4000$, $a = 5$ cm, $b = 6$ cm, $c = 2$ cm, and $I_0 = 50$ A.

Solution:

(a) We start by calculating the magnetic flux through the coil, noting that r , the distance from the wire varies from a to b

$$\begin{aligned}\Phi &= \int_S \mathbf{B} \cdot d\mathbf{s} = \int_a^b \hat{\mathbf{x}} \frac{\mu I}{2\pi r} \cdot \hat{\mathbf{x}} c \, dr = \frac{\mu c I}{2\pi} \ln\left(\frac{b}{a}\right) \\ V_{\text{emf}} &= -N \frac{d\Phi}{dt} = -\frac{\mu c N}{2\pi} \ln\left(\frac{b}{a}\right) \frac{dI}{dt} \\ &= \frac{\mu c N \omega I_0}{2\pi} \ln\left(\frac{b}{a}\right) \sin \omega t \quad (\text{V}).\end{aligned}$$

(b)

$$\begin{aligned}V_{\text{emf}} &= \frac{4000 \times 4\pi \times 10^{-7} \times 2 \times 10^{-2} \times 100 \times 2\pi \times 60 \times 50 \ln(6/5)}{2\pi} \sin 377t \\ &= 5.5 \sin 377t \quad (\text{V}).\end{aligned}$$