

Problem 7.1 The magnetic field of a wave propagating through a certain nonmagnetic material is given by

$$\mathbf{H} = \hat{\mathbf{z}} 30 \cos(10^8 t - 0.5y) \quad (\text{mA/m})$$

Find the following:

- (a) The direction of wave propagation.
- (b) The phase velocity.
- (c) The wavelength in the material.
- (d) The relative permittivity of the material.
- (e) The electric field phasor.

Solution:

- (a) Positive y-direction.
- (b) $\omega = 10^8 \text{ rad/s}$, $k = 0.5 \text{ rad/m}$.

$$u_p = \frac{\omega}{k} = \frac{10^8}{0.5} = 2 \times 10^8 \text{ m/s}.$$

- (c) $\lambda = 2\pi/k = 2\pi/0.5 = 12.6 \text{ m}$.

$$(d) \epsilon_r = \left(\frac{c}{u_p} \right)^2 = \left(\frac{3 \times 10^8}{2 \times 10^8} \right)^2 = 2.25.$$

- (e) From Eq. (7.39b),

$$\begin{aligned} \tilde{\mathbf{E}} &= -\eta \hat{\mathbf{k}} \times \tilde{\mathbf{H}}, \\ \eta &= \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{\epsilon_r}} = \frac{120\pi}{1.5} = 251.33 \quad (\Omega), \\ \hat{\mathbf{k}} &= \hat{\mathbf{y}}, \quad \text{and} \quad \tilde{\mathbf{H}} = \hat{\mathbf{z}} 30 e^{-j0.5y} \times 10^{-3} \quad (\text{A/m}). \end{aligned}$$

Hence,

$$\tilde{\mathbf{E}} = -251.33 \hat{\mathbf{y}} \times \hat{\mathbf{z}} 30 e^{-j0.5y} \times 10^{-3} = -\hat{\mathbf{x}} 7.54 e^{-j0.5y} \quad (\text{V/m}),$$

and

$$\mathbf{E}(y, t) = \Re(\tilde{\mathbf{E}} e^{j\omega t}) = -\hat{\mathbf{x}} 7.54 \cos(10^8 t - 0.5y) \quad (\text{V/m}).$$
