

7.11 The electric field of a uniform plane wave propagating in free space is given by

$$\tilde{\mathbf{E}} = (\hat{\mathbf{x}} + j\hat{\mathbf{y}})30e^{-j\pi z/6} \quad (\text{V/m})$$

Specify the modulus and direction of the electric field intensity at the $z = 0$ plane at $t = 0, 5$, and 10 ns.

Solution:

$$\begin{aligned} \mathbf{E}(z, t) &= \Re[\tilde{\mathbf{E}}e^{j\omega t}] \\ &= \Re[(\hat{\mathbf{x}} + j\hat{\mathbf{y}})30e^{-j\pi z/6}e^{j\omega t}] \\ &= \Re[(\hat{\mathbf{x}} + \hat{\mathbf{y}}e^{j\pi/2})30e^{-j\pi z/6}e^{j\omega t}] \\ &= \hat{\mathbf{x}}30\cos(\omega t - \pi z/6) + \hat{\mathbf{y}}30\cos(\omega t - \pi z/6 + \pi/2) \\ &= \hat{\mathbf{x}}30\cos(\omega t - \pi z/6) - \hat{\mathbf{y}}30\sin(\omega t - \pi z/6) \quad (\text{V/m}), \\ |\mathbf{E}| &= [E_x^2 + E_y^2]^{1/2} = 30 \quad (\text{V/m}), \\ \psi &= \tan^{-1}\left(\frac{E_y}{E_x}\right) = -(\omega t - \pi z/6). \end{aligned}$$

From

$$\begin{aligned} f &= \frac{c}{\lambda} = \frac{kc}{2\pi} = \frac{\pi/6 \times 3 \times 10^8}{2\pi} = 2.5 \times 10^7 \text{ Hz}, \\ \omega &= 2\pi f = 5\pi \times 10^7 \text{ rad/s}. \end{aligned}$$

At $z = 0$,

$$\psi = -\omega t = -5\pi \times 10^7 t = \begin{cases} 0 & \text{at } t = 0, \\ -0.25\pi = -45^\circ & \text{at } t = 5 \text{ ns}, \\ -0.5\pi = -90^\circ & \text{at } t = 10 \text{ ns}. \end{cases}$$

Therefore, the wave is LHC polarized.
