

Problem 7.12 The electric field of an elliptically polarized plane wave is given by

$$\mathbf{E}(z, t) = [-\hat{\mathbf{x}} 10 \sin(\omega t - kz - 60^\circ) + \hat{\mathbf{y}} 30 \cos(\omega t - kz)] \quad (\text{V/m})$$

Determine the following:

- (a) The polarization angles (γ, χ) .
- (b) The direction of rotation.

Solution:

(a)

$$\begin{aligned} \mathbf{E}(z, t) &= [-\hat{\mathbf{x}} 10 \sin(\omega t - kz - 60^\circ) + \hat{\mathbf{y}} 30 \cos(\omega t - kz)] \\ &= [\hat{\mathbf{x}} 10 \cos(\omega t - kz + 30^\circ) + \hat{\mathbf{y}} 30 \cos(\omega t - kz)] \quad (\text{V/m}). \end{aligned}$$

Phasor form:

$$\tilde{\mathbf{E}} = (\hat{\mathbf{x}} 10 e^{j30^\circ} + \hat{\mathbf{y}} 30) e^{-jkz}.$$

Since δ is defined as the phase of E_y relative to that of E_x ,

$$\delta = -30^\circ,$$

$$\psi_0 = \tan^{-1} \left(\frac{30}{10} \right) = 71.56^\circ,$$

$$\tan 2\gamma = (\tan 2\psi_0) \cos \delta = -0.65 \quad \text{or} \quad \gamma = 73.5^\circ,$$

$$\sin 2\chi = (\sin 2\psi_0) \sin \delta = -0.40 \quad \text{or} \quad \chi = -8.73^\circ.$$

(b) Since $\chi < 0$, the wave is right-hand elliptically polarized.
