

Problem 7.13 Compare the polarization states of each of the following pairs of plane waves:

- (a) Wave 1: $\mathbf{E}_1 = \hat{\mathbf{x}} 2 \cos(\omega t - kz) + \hat{\mathbf{y}} 2 \sin(\omega t - kz)$.
Wave 2: $\mathbf{E}_2 = \hat{\mathbf{x}} 2 \cos(\omega t + kz) + \hat{\mathbf{y}} 2 \sin(\omega t + kz)$.
- (b) Wave 1: $\mathbf{E}_1 = \hat{\mathbf{x}} 2 \cos(\omega t - kz) - \hat{\mathbf{y}} 2 \sin(\omega t - kz)$.
Wave 2: $\mathbf{E}_2 = \hat{\mathbf{x}} 2 \cos(\omega t + kz) - \hat{\mathbf{y}} 2 \sin(\omega t + kz)$.

Solution:

(a)

$$\begin{aligned}\mathbf{E}_1 &= \hat{\mathbf{x}} 2 \cos(\omega t - kz) + \hat{\mathbf{y}} 2 \sin(\omega t - kz) \\ &= \hat{\mathbf{x}} 2 \cos(\omega t - kz) + \hat{\mathbf{y}} 2 \cos(\omega t - kz - \pi/2), \\ \tilde{\mathbf{E}}_1 &= \hat{\mathbf{x}} 2 e^{-jkz} + \hat{\mathbf{y}} 2 e^{-jkz} e^{-j\pi/2}, \\ \psi_0 &= \tan^{-1} \left(\frac{ay}{ax} \right) = \tan^{-1} 1 = 45^\circ, \\ \delta &= -\pi/2.\end{aligned}$$

Hence, wave 1 is RHC.

Similarly,

$$\tilde{\mathbf{E}}_2 = \hat{\mathbf{x}} 2 e^{jkz} + \hat{\mathbf{y}} 2 e^{jkz} e^{-j\pi/2}.$$

Wave 2 has the same magnitude and phases as wave 1 except that its direction is along $-\hat{\mathbf{z}}$ instead of $+\hat{\mathbf{z}}$. Hence, the locus of rotation of \mathbf{E} will match the left hand instead of the right hand. Thus, wave 2 is LHC.

(b)

$$\begin{aligned}\mathbf{E}_1 &= \hat{\mathbf{x}} 2 \cos(\omega t - kz) - \hat{\mathbf{y}} 2 \sin(\omega t - kz), \\ \tilde{\mathbf{E}}_1 &= \hat{\mathbf{x}} 2 e^{-jkz} + \hat{\mathbf{y}} 2 e^{-jkz} e^{j\pi/2}.\end{aligned}$$

Wave 1 is LHC.

$$\tilde{\mathbf{E}}_2 = \hat{\mathbf{x}} 2 e^{jkz} + \hat{\mathbf{y}} 2 e^{jkz} e^{j\pi/2}.$$

Reversal of direction of propagation (relative to wave 1) makes wave 2 RHC.
