

Problem 7.17 In a medium characterized by $\epsilon_r = 9$, $\mu_r = 1$, and $\sigma = 0.1$ S/m, determine the phase angle by which the magnetic field leads the electric field at 100 MHz.

Solution: The phase angle by which the magnetic field leads the electric field is $-\theta_\eta$ where θ_η is the phase angle of η_c .

$$\frac{\sigma}{\omega\epsilon} = \frac{0.1 \times 36\pi}{2\pi \times 10^8 \times 10^{-9} \times 9} = 2.$$

Hence, quasi-conductor.

$$\begin{aligned}\eta_c &= \sqrt{\frac{\mu}{\epsilon'}} \left(1 - j\frac{\epsilon''}{\epsilon'}\right)^{-1/2} = \frac{120\pi}{\sqrt{\epsilon_r}} \left(1 - j\frac{\sigma}{\omega\epsilon_0\epsilon_r}\right)^{-1/2} \\ &= 125.67(1 - j2)^{-1/2} = 71.49 + j44.18 = 84.04 \angle 31.72^\circ.\end{aligned}$$

Therefore $\theta_\eta = 31.72^\circ$.

Since $\mathbf{H} = (1/\eta_c)\hat{\mathbf{k}} \times \mathbf{E}$, \mathbf{H} leads \mathbf{E} by $-\theta_\eta$, or by -31.72° . In other words, \mathbf{H} lags \mathbf{E} by 31.72° .
