

Problem 7.2 Write general expressions for the electric and magnetic fields of a 1-GHz sinusoidal plane wave traveling in the +y-direction in a lossless nonmagnetic medium with relative permittivity $\epsilon_r = 9$. The electric field is polarized along the x-direction, its peak value is 6 V/m, and its intensity is 4 V/m at $t = 0$ and $y = 2$ cm.

Solution: For $f = 1$ GHz, $\mu_r = 1$, and $\epsilon_r = 9$,

$$\omega = 2\pi f = 2\pi \times 10^9 \text{ rad/s},$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{\lambda_0} \sqrt{\epsilon_r} = \frac{2\pi f}{c} \sqrt{\epsilon_r} = \frac{2\pi \times 10^9}{3 \times 10^8} \sqrt{9} = 20\pi \text{ rad/m},$$

$$\mathbf{E}(y, t) = \hat{\mathbf{x}} 6 \cos(2\pi \times 10^9 t - 20\pi y + \phi_0) \quad (\text{V/m}).$$

At $t = 0$ and $y = 2$ cm, $E = 4$ V/m:

$$4 = 6 \cos(-20\pi \times 2 \times 10^{-2} + \phi_0) = 6 \cos(-0.4\pi + \phi_0).$$

Hence,

$$\phi_0 - 0.4\pi = \cos^{-1}\left(\frac{4}{6}\right) = 0.84 \text{ rad},$$

which gives

$$\phi_0 = 2.1 \text{ rad} = 120.19^\circ$$

and

$$\mathbf{E}(y, t) = \hat{\mathbf{x}} 6 \cos(2\pi \times 10^9 t - 20\pi y + 120.19^\circ) \quad (\text{V/m}).$$
