

Problem 7.22 The electric field of a plane wave propagating in a nonmagnetic medium is given by

$$\mathbf{E} = \hat{\mathbf{z}} 25 e^{-30x} \cos(2\pi \times 10^9 t - 40x) \quad (\text{V/m})$$

Obtain the corresponding expression for \mathbf{H} .

Solution: From the given expression for \mathbf{E} ,

$$\omega = 2\pi \times 10^9 \quad (\text{rad/s}),$$

$$\alpha = 30 \quad (\text{Np/m}),$$

$$\beta = 40 \quad (\text{rad/m}).$$

From (7.65a) and (7.65b),

$$\alpha^2 - \beta^2 = -\omega^2 \mu \epsilon' = -\omega^2 \mu_0 \epsilon_0 \epsilon'_r = -\frac{\omega^2}{c^2} \epsilon'_r,$$

$$2\alpha\beta = \omega^2 \mu \epsilon'' = \frac{\omega^2}{c^2} \epsilon''_r.$$

Using the above values for ω , α , and β , we obtain the following:

$$\epsilon'_r = 1.6,$$

$$\epsilon''_r = 5.47.$$

$$\begin{aligned} \eta_c &= \sqrt{\frac{\mu}{\epsilon'}} \left(1 - j \frac{\epsilon''_r}{\epsilon'_r} \right)^{-1/2} \\ &= \frac{\eta_0}{\sqrt{\epsilon'_r}} \left(1 - j \frac{\epsilon''_r}{\epsilon'_r} \right)^{-1/2} = \frac{377}{\sqrt{1.6}} \left(1 - j \frac{5.47}{1.6} \right)^{-1/2} = 157.9 e^{j36.85^\circ} \quad (\Omega). \end{aligned}$$

$$\tilde{\mathbf{E}} = \hat{\mathbf{z}} 25 e^{-30x} e^{-j40x},$$

$$\tilde{\mathbf{H}} = \frac{1}{\eta_c} \hat{\mathbf{k}} \times \tilde{\mathbf{E}} = \frac{1}{157.9 e^{j36.85^\circ}} \hat{\mathbf{x}} \times \hat{\mathbf{z}} 25 e^{-30x} e^{-j40x} = -\hat{\mathbf{y}} 0.16 e^{-30x} e^{-40x} e^{-j36.85^\circ},$$

$$\mathbf{H} = \Re\{\tilde{\mathbf{H}} e^{j\omega t}\} = -\hat{\mathbf{y}} 0.16 e^{-30x} \cos(2\pi \times 10^9 t - 40x - 36.85^\circ) \quad (\text{A/m}).$$
