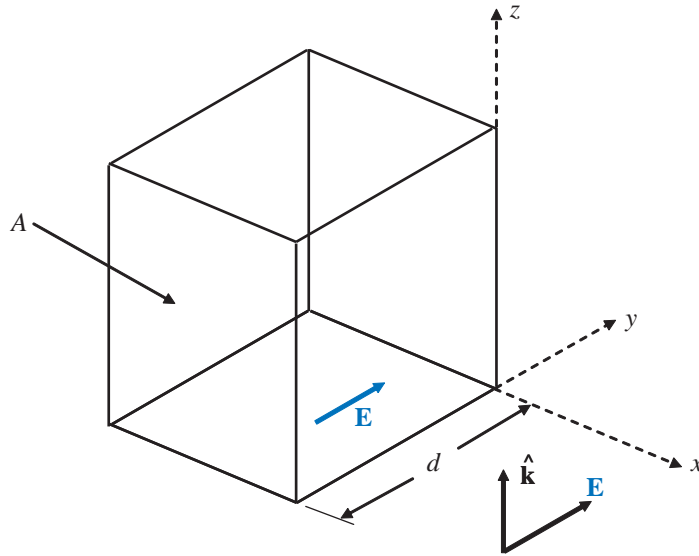


Problem 7.23 At 2 GHz, the conductivity of meat is on the order of 1 (S/m). When a material is placed inside a microwave oven and the field is activated, the presence of the electromagnetic fields in the conducting material causes energy dissipation in the material in the form of heat.

- (a) Develop an expression for the time-average power per mm^3 dissipated in a material of conductivity σ if the peak electric field in the material is E_0 .
- (b) Evaluate the result for an electric field $E_0 = 4 \times 10^4$ (V/m).

Solution:

(a) Let us consider a small volume of the material in the shape of a box of length d and cross sectional area A . Let us assume the microwave oven creates a wave traveling along the z direction with \mathbf{E} along y , as shown.



Along y , the \mathbf{E} field will create a voltage difference across the length of the box V , where

$$V = Ed.$$

Conduction current through the cross sectional area A is

$$I = JA = \sigma EA.$$

Hence, the instantaneous power is

$$\begin{aligned} P &= IV = \sigma E^2 (Ad) \\ &= \sigma E^2 v, \end{aligned}$$

where $\nu = Ad$ is the small volume under consideration. The power per mm^3 is obtained by setting $\nu = (10^{-3})^3$,

$$P' = \frac{P}{10^{-9}} = \sigma E^2 \times 10^{-9} \quad (\text{W/mm}^3).$$

As a time harmonic signal, $E = E_0 \cos \omega t$. The time average dissipated power is

$$\begin{aligned} P'_{\text{av}} &= \left[\frac{1}{T} \int_0^T \sigma E_0^2 \cos^2 \omega t \, dt \right] \times 10^{-9} \\ &= \frac{1}{2} \sigma E_0^2 \times 10^{-9} \quad (\text{W/mm}^3). \end{aligned}$$

(b)

$$P'_{\text{av}} = \frac{1}{2} \times 1 \times (4 \times 10^4) 2 \times 10^{-9} = 0.8 \quad (\text{W/mm}^3).$$
