

Problem 7.24 In a nonmagnetic, lossy, dielectric medium, a 300-MHz plane wave is characterized by the magnetic field phasor

$$\tilde{\mathbf{H}} = (\hat{\mathbf{x}} - j4\hat{\mathbf{z}})e^{-2y}e^{-j9y} \quad (\text{A/m})$$

Obtain time-domain expressions for the electric and magnetic field vectors.

Solution:

$$\tilde{\mathbf{E}} = -\eta_c \hat{\mathbf{k}} \times \tilde{\mathbf{H}}.$$

To find η_c , we need ϵ' and ϵ'' . From the given expression for $\tilde{\mathbf{H}}$,

$$\alpha = 2 \quad (\text{Np/m}),$$

$$\beta = 9 \quad (\text{rad/m}).$$

Also, we are given that $f = 300 \text{ MHz} = 3 \times 10^8 \text{ Hz}$. From (7.65a),

$$\begin{aligned} \alpha^2 - \beta^2 &= -\omega^2 \mu \epsilon', \\ 4 - 81 &= -(2\pi \times 3 \times 10^8)^2 \times 4\pi \times 10^{-7} \times \epsilon'_r \times \frac{10^{-9}}{36\pi}, \end{aligned}$$

whose solution gives

$$\epsilon'_r = 1.95.$$

Similarly, from (7.65b),

$$\begin{aligned} 2\alpha\beta &= \omega^2 \mu \epsilon'', \\ 2 \times 2 \times 9 &= (2\pi \times 3 \times 10^8)^2 \times 4\pi \times 10^{-7} \times \epsilon''_r \times \frac{10^{-9}}{36\pi}, \end{aligned}$$

which gives

$$\begin{aligned} \epsilon''_r &= 0.91. \\ \eta_c &= \sqrt{\frac{\mu}{\epsilon'}} \left(1 - j \frac{\epsilon''}{\epsilon'} \right)^{-1/2} \\ &= \frac{\eta_0}{\sqrt{\epsilon'_r}} \left(1 - j \frac{0.91}{1.95} \right)^{-1/2} = \frac{377}{\sqrt{1.95}} (0.93 + j0.21) = 256.9 e^{j12.6^\circ}. \end{aligned}$$

Hence,

$$\begin{aligned} \tilde{\mathbf{E}} &= -256.9 e^{j12.6^\circ} \hat{\mathbf{y}} \times (\hat{\mathbf{x}} - j4\hat{\mathbf{z}}) e^{-2y} e^{-j9y} \\ &= (\hat{\mathbf{x}} j4 + \hat{\mathbf{z}}) 256.9 e^{-2y} e^{-j9y} e^{j12.6^\circ} \\ &= (\hat{\mathbf{x}} 4e^{j\pi/2} + \hat{\mathbf{z}}) 256.9 e^{-2y} e^{-j9y} e^{j12.6^\circ}, \end{aligned}$$

$$\begin{aligned}
\mathbf{E} &= \Re\{\tilde{\mathbf{E}}e^{j\omega t}\} \\
&= \hat{\mathbf{x}}1.03 \times 10^3 e^{-2y} \cos(\omega t - 9y + 102.6^\circ) \\
&\quad + \hat{\mathbf{z}}256.9 e^{-2y} \cos(\omega t - 9y + 12.6^\circ) \quad (\text{V/m}), \\
\mathbf{H} &= \Re\{\tilde{\mathbf{H}}e^{j\omega t}\} \\
&= \Re\{(\hat{\mathbf{x}} + j4\hat{\mathbf{z}})e^{-2y}e^{-j9y}e^{j\omega t}\} \\
&= \hat{\mathbf{x}}e^{-2y} \cos(\omega t - 9y) + \hat{\mathbf{z}}4e^{-2y} \sin(\omega t - 9y) \quad (\text{A/m}).
\end{aligned}$$
