

Problem 7.29 The electric-field phasor of a uniform plane wave traveling downward in water is given by

$$\tilde{\mathbf{E}} = \hat{\mathbf{x}} 5e^{-0.2z} e^{-j0.2z} \quad (\text{V/m})$$

where $\hat{\mathbf{z}}$ is the downward direction and $z = 0$ is the water surface. If $\sigma = 4 \text{ S/m}$,

- (a) Obtain an expression for the average power density.
- (b) Determine the attenuation rate.
- (c) Determine the depth at which the power density has been reduced by 40 dB.

Solution:

- (a) Since $\alpha = \beta = 0.2$, the medium is a good conductor.

$$\eta_c = (1 + j) \frac{\alpha}{\sigma} = (1 + j) \frac{0.2}{4} = (1 + j)0.05 = 0.0707e^{j45^\circ} \quad (\Omega).$$

From Eq. (7.109),

$$\mathbf{S}_{\text{av}} = \hat{\mathbf{z}} \frac{|E_0|^2}{2|\eta_c|} e^{-2\alpha z} \cos \theta_\eta = \hat{\mathbf{z}} \frac{25}{2 \times 0.0707} e^{-0.4z} \cos 45^\circ = \hat{\mathbf{z}} 125e^{-0.4z} \quad (\text{W/m}^2).$$

- (b) $A = -8.68\alpha z = -8.68 \times 0.2z = -1.74z \text{ (dB)}.$
- (c) 40 dB is equivalent to 10^{-4} . Hence,

$$10^{-4} = e^{-2\alpha z} = e^{-0.4z}, \quad \ln(10^{-4}) = -0.4z,$$

or $z = 23.03 \text{ m}.$
