

Problem 7.3 The electric field phasor of a uniform plane wave is given by $\tilde{\mathbf{E}} = \hat{\mathbf{y}}10e^{j0.2z}$ (V/m). If the phase velocity of the wave is 1.5×10^8 m/s and the relative permeability of the medium is $\mu_r = 2.4$, find the following:

- (a) The wavelength.
- (b) The frequency f of the wave.
- (c) The relative permittivity of the medium.
- (d) The magnetic field $\mathbf{H}(z, t)$.

Solution:

(a) From $\tilde{\mathbf{E}} = \hat{\mathbf{y}}10e^{j0.2z}$ (V/m), we deduce that $k = 0.2$ rad/m. Hence,

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.2} = 10\pi = 31.42 \text{ m}.$$

(b)

$$f = \frac{u_p}{\lambda} = \frac{1.5 \times 10^8}{31.42} = 4.77 \times 10^6 \text{ Hz} = 4.77 \text{ MHz}.$$

(c) From

$$u_p = \frac{c}{\sqrt{\mu_r \epsilon_r}}, \quad \epsilon_r = \frac{1}{\mu_r} \left(\frac{c}{u_p} \right)^2 = \frac{1}{2.4} \left(\frac{3}{1.5} \right)^2 = 1.67.$$

(d)

$$\eta = \sqrt{\frac{\mu}{\epsilon}} \simeq 120\pi \sqrt{\frac{\mu_r}{\epsilon_r}} = 120\pi \sqrt{\frac{2.4}{1.67}} = 451.94 \quad (\Omega),$$

$$\tilde{\mathbf{H}} = \frac{1}{\eta} (-\hat{\mathbf{z}}) \times \tilde{\mathbf{E}} = \frac{1}{\eta} (-\hat{\mathbf{z}}) \times \hat{\mathbf{y}}10e^{j0.2z} = \hat{\mathbf{x}}22.13e^{j0.2z} \quad (\text{mA/m}),$$

$$\mathbf{H}(z, t) = \hat{\mathbf{x}}22.13 \cos(\omega t + 0.2z) \quad (\text{mA/m}),$$

with $\omega = 2\pi f = 9.54\pi \times 10^6$ rad/s.
