Problem 7.34 Repeat Problem 7.33 for a wave traveling in a lossy medium in which

$$\mathbf{E} = \hat{\mathbf{x}}\,100e^{-20y}\cos(2\pi \times 10^9 t - 40y) \quad \text{(V/m)}$$
$$\mathbf{H} = -\hat{\mathbf{z}}\,0.64e^{-20y}\cos(2\pi \times 10^9 t - 40y - 36.85^\circ) \quad \text{(A/m)}$$

The box has dimensions $a = 1$ cm, $b = 2$ cm, and $c = 0.5$ cm.

Solution:

(a)

$$\begin{aligned}\mathbf{S}(t) &= \mathbf{E} \times \mathbf{H}\\ &= \hat{\mathbf{x}}\,100e^{-20y}\cos(2\pi \times 10^9 t - 40y)\\ &\quad \times (-\hat{\mathbf{z}}\,0.64)e^{-20y}\cos(2\pi \times 10^9 t - 40y - 36.85^\circ)\\ &= \hat{\mathbf{y}}\,64e^{-40y}\cos(2\pi \times 10^9 t - 40y)\cos(2\pi \times 10^9 t - 40y - 36.85^\circ).\end{aligned}$$

Using the identity $\cos\theta\cos\phi = \frac{1}{2}[\cos(\theta+\phi) + \cos(\theta-\phi)]$,

$$\begin{aligned}S(t) &= \frac{64}{2}e^{-40y}[\cos(4\pi \times 10^9 t - 80y - 36.85^\circ) + \cos 36.85^\circ],\\ P(t) &= S(t)\,A|_{y=0} - S(t)\,A|_{y=b}\\ &= 32ac\{[\cos(4\pi \times 10^9 t - 36.85^\circ) + \cos 36.85^\circ]\\ &\quad - e^{-40b}[\cos(4\pi \times 10^9 t - 80y - 36.85^\circ) + \cos 36.85^\circ]\}.\end{aligned}$$

(b)

$$P_{\text{av}} = \frac{1}{T}\int_0^T P(t)\,dt = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} P(t)\,dt.$$

The average of $\cos(\omega t + \theta)$ over a period T is equal to zero, regardless of the value of θ. Hence,

$$P_{\text{av}} = 32ac(1 - e^{-40b})\cos 36.85^\circ.$$

With $a = 1$ cm, $b = 2$ cm, and $c = 0.5$ cm,

$$P_{\text{av}} = 7.05 \times 10^{-4} \quad \text{(W)}.$$

This is the average power absorbed by the lossy material in the box.