

Problem 7.4 The electric field of a plane wave propagating in a nonmagnetic material is given by

$$\mathbf{E} = [\hat{\mathbf{y}}3 \sin(\pi \times 10^7 t - 0.2\pi x) + \hat{\mathbf{z}}4 \cos(\pi \times 10^7 t - 0.2\pi x)] \quad (\text{V/m})$$

Determine

- (a) The wavelength.
- (b) ϵ_r .
- (c) \mathbf{H} .

Solution:

- (a) Since $k = 0.2\pi$,

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.2\pi} = 10 \text{ m}.$$

- (b)

$$u_p = \frac{\omega}{k} = \frac{\pi \times 10^7}{0.2\pi} = 5 \times 10^7 \text{ m/s}.$$

But

$$u_p = \frac{c}{\sqrt{\epsilon_r}}.$$

Hence,

$$\epsilon_r = \left(\frac{c}{u_p} \right)^2 = \left(\frac{3 \times 10^8}{5 \times 10^7} \right)^2 = 36.$$

- (c)

$$\begin{aligned} \mathbf{H} &= \frac{1}{\eta} \hat{\mathbf{k}} \times \mathbf{E} = \frac{1}{\eta} \hat{\mathbf{x}} \times [\hat{\mathbf{y}}3 \sin(\pi \times 10^7 t - 0.2\pi x) + \hat{\mathbf{z}}4 \cos(\pi \times 10^7 t - 0.2\pi x)] \\ &= \hat{\mathbf{z}} \frac{3}{\eta} \sin(\pi \times 10^7 t - 0.2\pi x) - \hat{\mathbf{y}} \frac{4}{\eta} \cos(\pi \times 10^7 t - 0.2\pi x) \quad (\text{A/m}), \end{aligned}$$

with

$$\eta = \frac{\eta_0}{\sqrt{\epsilon_r}} \simeq \frac{120\pi}{6} = 20\pi = 62.83 \quad (\Omega).$$
