

Problem 7.7 A 60-MHz plane wave traveling in the $-x$ -direction in dry soil with relative permittivity $\epsilon_r = 4$ has an electric field polarized along the z -direction. Assuming dry soil to be approximately lossless, and given that the magnetic field has a peak value of 10 (mA/m) and that its value was measured to be 7 (mA/m) at $t = 0$ and $x = -0.75$ m, develop complete expressions for the wave's electric and magnetic fields.

Solution: For $f = 60 \text{ MHz} = 6 \times 10^7 \text{ Hz}$, $\epsilon_r = 4$, $\mu_r = 1$,

$$k = \frac{\omega}{c} \sqrt{\epsilon_r} = \frac{2\pi \times 6 \times 10^7}{3 \times 10^8} \sqrt{4} = 0.8\pi \quad (\text{rad/m}).$$

Given that \mathbf{E} points along $\hat{\mathbf{z}}$ and wave travel is along $-\hat{\mathbf{x}}$, we can write

$$\mathbf{E}(x, t) = \hat{\mathbf{z}} E_0 \cos(2\pi \times 60 \times 10^6 t + 0.8\pi x + \phi_0) \quad (\text{V/m})$$

where E_0 and ϕ_0 are unknown constants at this time. The intrinsic impedance of the medium is

$$\eta = \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{120\pi}{2} = 60\pi \quad (\Omega).$$

With \mathbf{E} along $\hat{\mathbf{z}}$ and $\hat{\mathbf{k}}$ along $-\hat{\mathbf{x}}$, (7.39) gives

$$\mathbf{H} = \frac{1}{\eta} \hat{\mathbf{k}} \times \mathbf{E}$$

or

$$\mathbf{H}(x, t) = \hat{\mathbf{y}} \frac{E_0}{\eta} \cos(1.2\pi \times 10^8 t + 0.8\pi x + \phi_0) \quad (\text{A/m}).$$

Hence,

$$\begin{aligned} \frac{E_0}{\eta} &= 10 \quad (\text{mA/m}) \\ E_0 &= 10 \times 60\pi \times 10^{-3} = 0.6\pi \quad (\text{V/m}). \end{aligned}$$

Also,

$$H(-0.75 \text{ m}, 0) = 7 \times 10^{-3} = 10 \cos(-0.8\pi \times 0.75 + \phi_0) \times 10^{-3}$$

which leads to $\phi_0 = 153.6^\circ$.

Hence,

$$\mathbf{E}(x, t) = \hat{\mathbf{z}} 0.6\pi \cos(1.2\pi \times 10^8 t + 0.8\pi x + 153.6^\circ) \quad (\text{V/m}).$$

$$\mathbf{H}(x, t) = \hat{\mathbf{y}} 10 \cos(1.2\pi \times 10^8 t + 0.8\pi x + 153.6^\circ) \quad (\text{mA/m}).$$
